

Kinematics of a Particle

12

CHAPTER OBJECTIVES

- To introduce the concepts of position, displacement, velocity, and acceleration.
- To study particle motion along a straight line and represent this motion graphically.
- To investigate particle motion along a curved path using different coordinate systems.
- To present an analysis of dependent motion of two particles.
- To examine the principles of relative motion of two particles using translating axes.

12.1 Introduction

Mechanics is a branch of the physical sciences that is concerned with the state of rest or motion of bodies subjected to the action of forces. Engineering mechanics is divided into two areas of study, namely, statics and dynamics. *Statics* is concerned with the equilibrium of a body that is either at rest or moves with constant velocity. Here we will consider *dynamics*, which deals with the accelerated motion of a body. The subject of dynamics will be presented in two parts: *kinematics*, which treats only the geometric aspects of the motion, and *kinetics*, which is the analysis of the forces causing the motion. To develop these principles, the dynamics of a particle will be discussed first, followed by topics in rigid-body dynamics in two and then three dimensions.

Historically, the principles of dynamics developed when it was possible to make an accurate measurement of time. Galileo Galilei (1564–1642) was one of the first major contributors to this field. His work consisted of experiments using pendulums and falling bodies. The most significant contributions in dynamics, however, were made by Isaac Newton (1642–1727), who is noted for his formulation of the three fundamental laws of motion and the law of universal gravitational attraction. Shortly after these laws were postulated, important techniques for their application were developed by Euler, D'Alembert, Lagrange, and others.

There are many problems in engineering whose solutions require application of the principles of dynamics. Typically the structural design of any vehicle, such as an automobile or airplane, requires consideration of the motion to which it is subjected. This is also true for many mechanical devices, such as motors, pumps, movable tools, industrial manipulators, and machinery. Furthermore, predictions of the motions of artificial satellites, projectiles, and spacecraft are based on the theory of dynamics. With further advances in technology, there will be an even greater need for knowing how to apply the principles of this subject.

Problem Solving. Dynamics is considered to be more involved than statics since both the forces applied to a body and its motion must be taken into account. Also, many applications require using calculus, rather than just algebra and trigonometry. In any case, the most effective way of learning the principles of dynamics is *to solve problems*. To be successful at this, it is necessary to present the work in a logical and orderly manner as suggested by the following sequence of steps:

1. Read the problem carefully and try to correlate the actual physical situation with the theory you have studied.
2. Draw any necessary diagrams and tabulate the problem data.
3. Establish a coordinate system and apply the relevant principles, generally in mathematical form.
4. Solve the necessary equations algebraically as far as practical; then, use a consistent set of units and complete the solution numerically. Report the answer with no more significant figures than the accuracy of the given data.
5. Study the answer using technical judgment and common sense to determine whether or not it seems reasonable.
6. Once the solution has been completed, review the problem. Try to think of other ways of obtaining the same solution.

In applying this general procedure, do the work as neatly as possible. Being neat generally stimulates clear and orderly thinking, and vice versa.

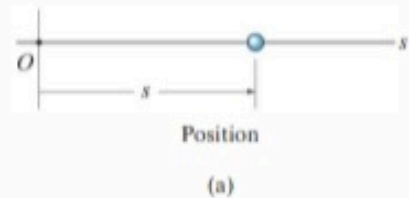
12.2 Rectilinear Kinematics: Continuous Motion

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We will begin our study of dynamics by discussing the kinematics of a particle that moves along a rectilinear or straight line path. Recall that a *particle* has a mass but negligible size and shape. Therefore we must limit application to those objects that have dimensions that are of no consequence in the analysis of the motion. In most problems, we will be interested in bodies of finite size, such as rockets, projectiles, or vehicles. Each of these objects can be considered as a particle, as long as the motion is characterized by the motion of its mass center and any rotation of the body is neglected.

Rectilinear Kinematics. The kinematics of a particle is characterized by specifying, at any given instant, the particle's position, velocity, and acceleration.

Position. The straight-line path of a particle will be defined using a single coordinate axis s , Fig. 12-1a. The origin O on the path is a fixed point, and from this point the *position coordinate* s is used to specify the location of the particle at any given instant. The magnitude of s is the distance from O to the particle, usually measured in meters (m) or feet (ft), and the sense of direction is defined by the algebraic sign on s . Although the choice is arbitrary, in this case s is positive since the coordinate axis is positive to the right of the origin. Likewise, it is negative if the particle is located to the left of O . Realize that position is a vector quantity since it has both magnitude and direction. Here, however, it is being represented by the algebraic scalar s since the direction always remains along the coordinate axis.



Displacement. The *displacement* of the particle is defined as the *change* in its *position*. For example, if the particle moves from one point to another, Fig. 12-1b, the displacement is

$$\Delta s = s' - s$$

In this case Δs is *positive* since the particle's final position is to the *right* of its initial position, i.e., $s' > s$. Likewise, if the final position were to the *left* of its initial position, Δs would be *negative*.

The displacement of a particle is also a *vector quantity*, and it should be distinguished from the distance the particle travels. Specifically, the *distance traveled* is a *positive scalar* that represents the total length of path over which the particle travels.

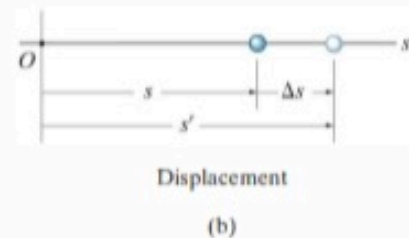


Fig. 12-1

Velocity. If the particle moves through a displacement Δs during the time interval Δt , the *average velocity* of the particle during this time interval is

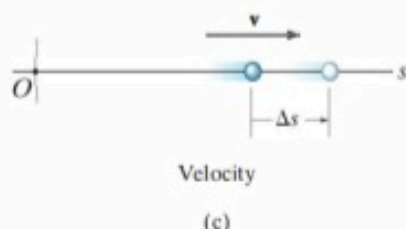
$$v_{\text{avg}} = \frac{\Delta s}{\Delta t}$$

If we take smaller and smaller values of Δt , the magnitude of Δs becomes smaller and smaller. Consequently, the *instantaneous velocity* is a vector defined as $v = \lim_{\Delta t \rightarrow 0} (\Delta s / \Delta t)$, or

($\pm \rightarrow$)

$$v = \frac{ds}{dt}$$

(12-1)



Since Δt or dt is always positive, the sign used to define the *sense* of the velocity is the same as that of Δs or ds . For example, if the particle is moving to the *right*, Fig. 12-1c, the velocity is *positive*; whereas if it is moving to the *left*, the velocity is *negative*. (This is emphasized here by the arrow written at the left of Eq. 12-1.) The *magnitude* of the velocity is known as the *speed*, and it is generally expressed in units of m/s or ft/s.

Occasionally, the term “average speed” is used. The *average speed* is always a positive scalar and is defined as the total distance traveled by a particle, s_T , divided by the elapsed time Δt ; i.e.,

$$(v_{\text{sp}})_{\text{avg}} = \frac{s_T}{\Delta t}$$

For example, the particle in Fig. 12-1d travels along the path of length s_T in time Δt , so its average speed is $(v_{\text{sp}})_{\text{avg}} = s_T / \Delta t$, but its average velocity is $v_{\text{avg}} = -\Delta s / \Delta t$.

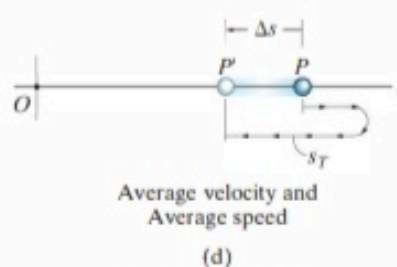


Fig. 12-1 (cont.)

Acceleration. Provided the velocity of the particle is known at two points, the *average acceleration* of the particle during the time interval Δt is defined as

$$a_{\text{avg}} = \frac{\Delta v}{\Delta t}$$

Here Δv represents the difference in the velocity during the time interval Δt , i.e., $\Delta v = v' - v$, Fig. 12-1e.

The *instantaneous acceleration* at time t is a vector that is found by taking smaller and smaller values of Δt and corresponding smaller and smaller values of Δv , so that $a = \lim_{\Delta t \rightarrow 0} (\Delta v / \Delta t)$, or

$$(\pm) \quad a = \frac{dv}{dt} \quad (12-2)$$

Substituting Eq. 12-1 into this result, we can also write

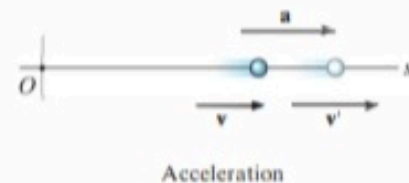
$$(\pm) \quad a = \frac{d^2s}{dt^2}$$

Both the average and instantaneous acceleration can be either positive or negative. In particular, when the particle is *slowing down*, or its speed is decreasing, the particle is said to be *decelerating*. In this case, v' in Fig. 12-1f is *less* than v , and so $\Delta v = v' - v$ will be negative. Consequently, a will also be negative, and therefore it will act to the *left*, in the *opposite sense* to v . Also, note that when the *velocity is constant*, the *acceleration is zero* since $\Delta v = v - v = 0$. Units commonly used to express the magnitude of acceleration are m/s^2 or ft/s^2 .

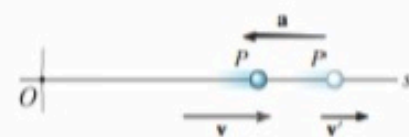
Finally, an important differential relation involving the displacement, velocity, and acceleration along the path may be obtained by eliminating the time differential dt between Eqs. 12-1 and 12-2, which gives

$$(\pm) \quad a \, ds = v \, dv \quad (12-3)$$

Although we have now produced three important kinematic equations, realize that the above equation is not independent of Eqs. 12-1 and 12-2.



(c)



(f)

Constant Acceleration, $a = a_c$. When the acceleration is constant, each of the three kinematic equations $a_c = dv/dt$, $v = ds/dt$, and $a_c ds = v dv$ can be integrated to obtain formulas that relate a_c , v , s , and t .

Velocity as a Function of Time. Integrate $a_c = dv/dt$, assuming that initially $v = v_0$ when $t = 0$.

$$\int_{v_0}^v dv = \int_0^t a_c dt$$

($\rightarrow$)

$$v = v_0 + a_c t$$

Constant Acceleration

(12-4)

Position as a Function of Time. Integrate $v = ds/dt = v_0 + a_c t$, assuming that initially $s = s_0$ when $t = 0$.

$$\int_{s_0}^s ds = \int_0^t (v_0 + a_c t) dt$$

($\rightarrow$)

$$s = s_0 + v_0 t + \frac{1}{2} a_c t^2$$

Constant Acceleration

(12-5)

Velocity as a Function of Position. Either solve for t in Eq. 12-4 and substitute into Eq. 12-5, or integrate $v dv = a_c ds$, assuming that initially $v = v_0$ at $s = s_0$.

$$\int_{v_0}^v v dv = \int_{s_0}^s a_c ds$$

($\rightarrow$)

$$v^2 = v_0^2 + 2a_c(s - s_0)$$

Constant Acceleration

(12-6)

The algebraic signs of s_0 , v_0 , and a_c , used in the above three equations, are determined from the positive direction of the s axis as indicated by the arrow written at the left of each equation. Remember that these equations are useful *only when the acceleration is constant and when* $t = 0$, $s = s_0$, $v = v_0$. A typical example of constant accelerated motion occurs when a body falls freely toward the earth. If air resistance is neglected and the distance of fall is short, then the *downward* acceleration of the body when it is close to the earth is constant and approximately 9.81 m/s^2 or 32.2 ft/s^2 . The proof of this is given in Example 13.2.

Important Points

- Dynamics is concerned with bodies that have accelerated motion.
- Kinematics is a study of the geometry of the motion.
- Kinetics is a study of the forces that cause the motion.
- Rectilinear kinematics refers to straight-line motion.
- Speed refers to the magnitude of velocity.
- Average speed is the total distance traveled divided by the total time. This is different from the average velocity, which is the displacement divided by the time.
- A particle that is slowing down is decelerating.
- A particle can have an acceleration and yet have zero velocity.
- The relationship $a ds = v dv$ is derived from $a = dv/dt$ and $v = ds/dt$, by eliminating dt .



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During the time this rocket undergoes rectilinear motion, its altitude as a function of time can be measured and expressed as $s = s(t)$. Its velocity can then be found using $v = ds/dt$, and its acceleration can be determined from $a = dv/dt$.

Procedure for Analysis

Coordinate System.

- Establish a position coordinate s along the path and specify its *fixed origin* and positive direction.
- Since motion is along a straight line, the vector quantities position, velocity, and acceleration can be represented as algebraic scalars. For analytical work the sense of s , v , and a is then defined by their *algebraic signs*.
- The positive sense for each of these scalars can be indicated by an arrow shown alongside each kinematic equation as it is applied.

Kinematic Equations.

- If a relation is known between any *two* of the four variables a , v , s and t , then a third variable can be obtained by using one of the kinematic equations, $a = dv/dt$, $v = ds/dt$ or $a ds = v dv$, since each equation relates all three variables.*
- Whenever integration is performed, it is important that the position and velocity be known at a given instant in order to evaluate either the constant of integration if an indefinite integral is used, or the limits of integration if a definite integral is used.
- Remember that Eqs. 12–4 through 12–6 have only limited use. These equations apply *only* when the *acceleration is constant* and the initial conditions are $s = s_0$ and $v = v_0$ when $t = 0$.

*Some standard differentiation and integration formulas are given in Appendix A.

The car in Fig. 12-2 moves in a straight line such that for a short time its velocity is defined by $v = (3t^2 + 2t)$ ft/s, where t is in seconds. Determine its position and acceleration when $t = 3$ s. When $t = 0$, $s = 0$.



Fig. 12-2

SOLUTION

Coordinate System. The position coordinate extends from the fixed origin O to the car, positive to the right.

Position. Since $v = f(t)$, the car's position can be determined from $v = ds/dt$, since this equation relates v , s , and t . Noting that $s = 0$ when $t = 0$, we have*

$$\begin{aligned}
 (\rightarrow) \quad v &= \frac{ds}{dt} = (3t^2 + 2t) \\
 \int_0^s ds &= \int_0^t (3t^2 + 2t) dt \\
 s \Big|_0^s &= t^3 + t^2 \Big|_0^t \\
 s &= t^3 + t^2
 \end{aligned}$$

When $t = 3$ s,

$$s = (3)^3 + (3)^2 = 36 \text{ ft} \quad \text{Ans.}$$

Acceleration. Since $v = f(t)$, the acceleration is determined from $a = dv/dt$, since this equation relates a , v , and t .

$$\begin{aligned}
 (\rightarrow) \quad a &= \frac{dv}{dt} = \frac{d}{dt}(3t^2 + 2t) \\
 &= 6t + 2
 \end{aligned}$$

When $t = 3$ s,

$$a = 6(3) + 2 = 20 \text{ ft/s}^2 \rightarrow \quad \text{Ans.}$$

NOTE: The formulas for constant acceleration *cannot* be used to solve this problem, because the acceleration is a function of time.

*The *same result* can be obtained by evaluating a constant of integration C rather than using definite limits on the integral. For example, integrating $ds = (3t^2 + 2t)dt$ yields $s = t^3 + t^2 + C$. Using the condition that at $t = 0$, $s = 0$, then $C = 0$.

EXAMPLE 12.2

A small projectile is fired vertically *downward* into a fluid medium with an initial velocity of 60 m/s. Due to the drag resistance of the fluid the projectile experiences a deceleration of $a = (-0.4v^3)$ m/s², where v is in m/s. Determine the projectile's velocity and position 4 s after it is fired.

SOLUTION

Coordinate System. Since the motion is downward, the position coordinate is positive downward, with origin located at O , Fig. 12-3.

Velocity. Here $a = f(v)$ and so we must determine the velocity as a function of time using $a = dv/dt$, since this equation relates v , a , and t . (Why not use $v = v_0 + a_c t$?) Separating the variables and integrating, with $v_0 = 60$ m/s when $t = 0$, yields

$$\begin{aligned}
 (+\downarrow) \quad a &= \frac{dv}{dt} = -0.4v^3 \\
 \int_{60 \text{ m/s}}^v \frac{dv}{-0.4v^3} &= \int_0^t dt \\
 -\frac{1}{0.4} \left(\frac{1}{-2} \right) \frac{1}{v^2} \bigg|_{60}^v &= t - 0 \\
 \frac{1}{0.8} \left[\frac{1}{v^2} - \frac{1}{(60)^2} \right] &= t \\
 v &= \left\{ \left[\frac{1}{(60)^2} + 0.8t \right]^{-1/2} \right\} \text{ m/s}
 \end{aligned}$$

Here the positive root is taken, since the projectile will continue to move downward. When $t = 4$ s,

$$v = 0.559 \text{ m/s} \downarrow \quad \text{Ans.}$$

Position. Knowing $v = f(t)$, we can obtain the projectile's position from $v = ds/dt$, since this equation relates s , v , and t . Using the initial condition $s = 0$, when $t = 0$, we have

$$\begin{aligned}
 (+\downarrow) \quad v &= \frac{ds}{dt} = \left[\frac{1}{(60)^2} + 0.8t \right]^{-1/2} \\
 \int_0^s ds &= \int_0^t \left[\frac{1}{(60)^2} + 0.8t \right]^{-1/2} dt \\
 s &= \frac{2}{0.8} \left[\frac{1}{(60)^2} + 0.8t \right]^{1/2} \bigg|_0^t \\
 s &= \frac{1}{0.4} \left\{ \left[\frac{1}{(60)^2} + 0.8t \right]^{1/2} - \frac{1}{60} \right\} \text{ m}
 \end{aligned}$$

When $t = 4$ s,

$$s = 4.43 \text{ m} \quad \text{Ans.}$$



Fig. 12-3

EXAMPLE 12.3

During a test a rocket travels upward at 75 m/s, and when it is 40 m from the ground its engine fails. Determine the maximum height s_B reached by the rocket and its speed just before it hits the ground. While in motion the rocket is subjected to a constant downward acceleration of 9.81 m/s^2 due to gravity. Neglect the effect of air resistance.

SOLUTION

Coordinate System. The origin O for the position coordinate s is taken at ground level with positive upward, Fig. 12-4.

Maximum Height. Since the rocket is traveling *upward*, $v_A = +75 \text{ m/s}$ when $t = 0$. At the maximum height $s = s_B$ the velocity $v_B = 0$. For the entire motion, the acceleration is $a_c = -9.81 \text{ m/s}^2$ (negative since it acts in the *opposite* sense to positive velocity or positive displacement). Since a_c is *constant* the rocket's position may be related to its velocity at the two points A and B on the path by using Eq. 12-6, namely,

$$\begin{aligned} (+\uparrow) \quad v_B^2 &= v_A^2 + 2a_c(s_B - s_A) \\ 0 &= (75 \text{ m/s})^2 + 2(-9.81 \text{ m/s}^2)(s_B - 40 \text{ m}) \\ s_B &= 327 \text{ m} \end{aligned}$$

Ans.

Velocity. To obtain the velocity of the rocket just before it hits the ground, we can apply Eq. 12-6 between points B and C , Fig. 12-4.

$$\begin{aligned} (+\uparrow) \quad v_C^2 &= v_B^2 + 2a_c(s_C - s_B) \\ &= 0 + 2(-9.81 \text{ m/s}^2)(0 - 327 \text{ m}) \\ v_C &= -80.1 \text{ m/s} = 80.1 \text{ m/s} \downarrow \end{aligned}$$

Ans.

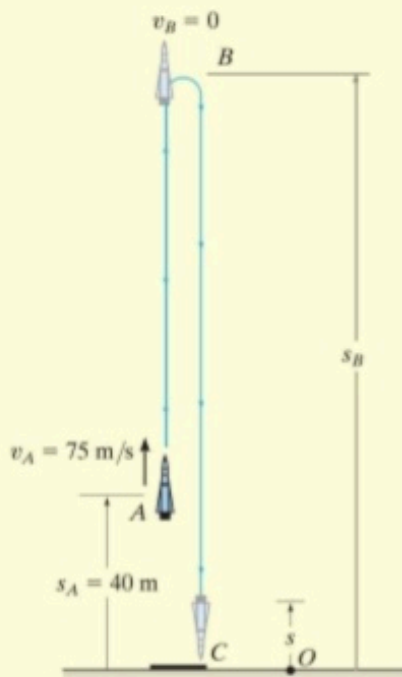
The negative root was chosen since the rocket is moving downward.

Similarly, Eq. 12-6 may also be applied between points A and C , i.e.,

$$\begin{aligned} (+\uparrow) \quad v_C^2 &= v_A^2 + 2a_c(s_C - s_A) \\ &= (75 \text{ m/s})^2 + 2(-9.81 \text{ m/s}^2)(0 - 40 \text{ m}) \\ v_C &= -80.1 \text{ m/s} = 80.1 \text{ m/s} \downarrow \end{aligned}$$

Ans.

NOTE: It should be realized that the rocket is subjected to a *deceleration* from A to B of 9.81 m/s^2 , and then from B to C it is *accelerated* at this rate. Furthermore, even though the rocket momentarily comes to *rest* at B ($v_B = 0$) the acceleration at B is still 9.81 m/s^2 downward!

**Fig. 12-4**

EXAMPLE 12.4

A metallic particle is subjected to the influence of a magnetic field as it travels downward through a fluid that extends from plate *A* to plate *B*, Fig. 12–5. If the particle is released from rest at the midpoint *C*, $s = 100$ mm, and the acceleration is $a = (4s) \text{ m/s}^2$, where s is in meters, determine the velocity of the particle when it reaches plate *B*, $s = 200$ mm, and the time it takes to travel from *C* to *B*.

SOLUTION

Coordinate System. As shown in Fig. 12–5, s is positive downward, measured from plate *A*.

Velocity. Since $a = f(s)$, the velocity as a function of position can be obtained by using $v dv = a ds$. Realizing that $v = 0$ at $s = 0.1$ m, we have

$$\begin{aligned}
 (+\downarrow) \quad v dv &= a ds \\
 \int_0^v v dv &= \int_{0.1 \text{ m}}^s 4s ds \\
 \left. \frac{1}{2} v^2 \right|_0^v &= \left. \frac{4}{2} s^2 \right|_{0.1 \text{ m}}^s \\
 v &= 2(s^2 - 0.01)^{1/2} \text{ m/s} \quad (1)
 \end{aligned}$$

At $s = 200 \text{ mm} = 0.2 \text{ m}$,

$$v_B = 0.346 \text{ m/s} = 346 \text{ mm/s} \downarrow \quad \text{Ans.}$$

The positive root is chosen since the particle is traveling downward, i.e., in the $+s$ direction.

Time. The time for the particle to travel from *C* to *B* can be obtained using $v = ds/dt$ and Eq. 1, where $s = 0.1 \text{ m}$ when $t = 0$. From Appendix A,

$$\begin{aligned}
 (+\downarrow) \quad ds &= v dt \\
 &= 2(s^2 - 0.01)^{1/2} dt \\
 \int_{0.1}^s \frac{ds}{(s^2 - 0.01)^{1/2}} &= \int_0^t 2 dt \\
 \ln(\sqrt{s^2 - 0.01} + s) \Big|_{0.1}^s &= 2t \Big|_0^t \\
 \ln(\sqrt{s^2 - 0.01} + s) + 2.303 &= 2t
 \end{aligned}$$

At $s = 0.2 \text{ m}$,

$$t = \frac{\ln(\sqrt{(0.2)^2 - 0.01} + 0.2) + 2.303}{2} = 0.658 \text{ s} \quad \text{Ans.}$$

Note: The formulas for constant acceleration cannot be used here because the acceleration changes with position, i.e., $a = 4s$.

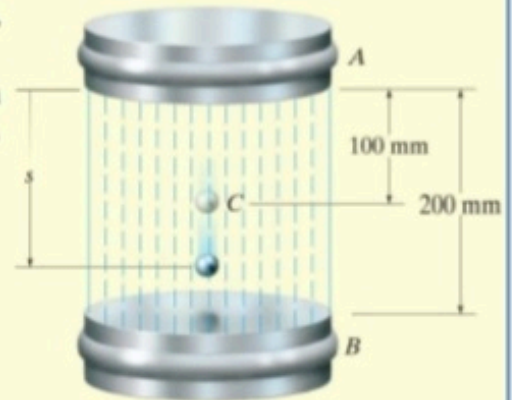


Fig. 12–5

EXAMPLE 12.5

A particle moves along a horizontal path with a velocity of $v = (3t^2 - 6t)$ m/s, where t is the time in seconds. If it is initially located at the origin O , determine the distance traveled in 3.5 s, and the particle's average velocity and average speed during the time interval.

SOLUTION

Coordinate System. Here positive motion is to the right, measured from the origin O , Fig. 12-6a.

Distance Traveled. Since $v = f(t)$, the position as a function of time may be found by integrating $v = ds/dt$ with $t = 0, s = 0$.

$$\begin{aligned}
 (\rightarrow) \quad ds &= v \, dt \\
 &= (3t^2 - 6t) \, dt \\
 \int_0^s ds &= \int_0^t (3t^2 - 6t) \, dt \\
 s &= (t^3 - 3t^2) \text{ m} \quad (1)
 \end{aligned}$$

In order to determine the distance traveled in 3.5 s, it is necessary to investigate the path of motion. If we consider a graph of the velocity function, Fig. 12-6b, then it reveals that for $0 < t < 2$ s the velocity is *negative*, which means the particle is traveling to the *left*, and for $t > 2$ s the velocity is *positive*, and hence the particle is traveling to the *right*. Also, note that $v = 0$ at $t = 2$ s. The particle's position when $t = 0$, $t = 2$ s, and $t = 3.5$ s can now be determined from Eq. 1. This yields

$$s|_{t=0} = 0 \quad s|_{t=2\text{ s}} = -4.0 \text{ m} \quad s|_{t=3.5\text{ s}} = 6.125 \text{ m}$$

The path is shown in Fig. 12-6a. Hence, the distance traveled in 3.5 s is

$$s_T = 4.0 + 4.0 + 6.125 = 14.125 \text{ m} = 14.1 \text{ m} \quad \text{Ans.}$$

Velocity. The *displacement* from $t = 0$ to $t = 3.5$ s is

$$\Delta s = s|_{t=3.5\text{ s}} - s|_{t=0} = 6.125 \text{ m} - 0 = 6.125 \text{ m}$$

and so the average velocity is

$$v_{\text{avg}} = \frac{\Delta s}{\Delta t} = \frac{6.125 \text{ m}}{3.5 \text{ s} - 0} = 1.75 \text{ m/s} \rightarrow \quad \text{Ans.}$$

The average speed is defined in terms of the *distance traveled* s_T . This positive scalar is

$$(v_{\text{sp}})_{\text{avg}} = \frac{s_T}{\Delta t} = \frac{14.125 \text{ m}}{3.5 \text{ s} - 0} = 4.04 \text{ m/s} \quad \text{Ans.}$$

Note: In this problem, the acceleration is $a = dv/dt = (6t - 6)$ m/s², which is not constant.

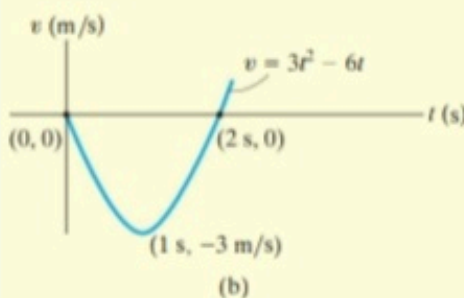
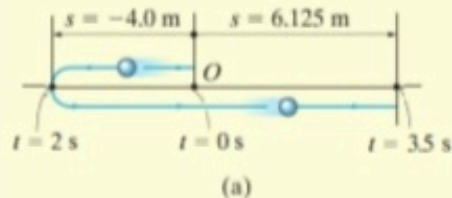


Fig. 12-6

F12-1. Initially, the car travels along a straight road with a speed of 35 m/s. If the brakes are applied and the speed of the car is reduced to 10 m/s in 15 s, determine the constant deceleration of the car.



F12-1

F12-2. A ball is thrown vertically upward with a speed of 15 m/s. Determine the time of flight when it returns to its original position.



F12-2

F12-3. A particle travels along a straight line with a velocity of $v = (4t - 3t^2)$ m/s, where t is in seconds. Determine the position of the particle when $t = 4$ s. $s = 0$ when $t = 0$.



F12-3

F12-4. A particle travels along a straight line with a speed $v = (0.5t^3 - 8t)$ m/s, where t is in seconds. Determine the acceleration of the particle when $t = 2$ s.



F12-4

F12-5. The position of the particle is given by $s = (2t^2 - 8t + 6)$ m, where t is in seconds. Determine the time when the velocity of the particle is zero, and the total distance traveled by the particle when $t = 3$ s.



F12-5

F12-6. A particle travels along a straight line with an acceleration of $a = (10 - 0.2s)$ m/s², where s is measured in meters. Determine the velocity of the particle when $s = 10$ m if $v = 5$ m/s at $s = 0$.



F12-6

F12-7. A particle moves along a straight line such that its acceleration is $a = (4t^2 - 2)$ m/s², where t is in seconds. When $t = 0$, the particle is located 2 m to the left of the origin, and when $t = 2$ s, it is 20 m to the left of the origin. Determine the position of the particle when $t = 4$ s.



F12-7

F12-8. A particle travels along a straight line with a velocity of $v = (20 - 0.05s^2)$ m/s, where s is in meters. Determine the acceleration of the particle at $s = 15$ m.



F12-8

•12-1. A car starts from rest and with constant acceleration achieves a velocity of 15 m/s when it travels a distance of 200 m. Determine the acceleration of the car and the time required.

12-2. A train starts from rest at a station and travels with a constant acceleration of 1 m/s^2 . Determine the velocity of the train when $t = 30 \text{ s}$ and the distance traveled during this time.

12-3. An elevator descends from rest with an acceleration of 5 ft/s^2 until it achieves a velocity of 15 ft/s. Determine the time required and the distance traveled.

*12-4. A car is traveling at 15 m/s, when the traffic light 50 m ahead turns yellow. Determine the required constant deceleration of the car and the time needed to stop the car at the light.

•12-5. A particle is moving along a straight line with the acceleration $a = (12t - 3t^{1/2}) \text{ ft/s}^2$, where t is in seconds. Determine the velocity and the position of the particle as a function of time. When $t = 0$, $v = 0$ and $s = 15 \text{ ft}$.

12-6. A ball is released from the bottom of an elevator which is traveling upward with a velocity of 6 ft/s. If the ball strikes the bottom of the elevator shaft in 3 s, determine the height of the elevator from the bottom of the shaft at the instant the ball is released. Also, find the velocity of the ball when it strikes the bottom of the shaft.

12-7. A car has an initial speed of 25 m/s and a constant deceleration of 3 m/s^2 . Determine the velocity of the car when $t = 4 \text{ s}$. What is the displacement of the car during the 4-s time interval? How much time is needed to stop the car?

*12-8. If a particle has an initial velocity of $v_0 = 12 \text{ ft/s}$ to the right, at $s_0 = 0$, determine its position when $t = 10 \text{ s}$, if $a = 2 \text{ ft/s}^2$ to the left.

•12-9. The acceleration of a particle traveling along a straight line is $a = k/v$, where k is a constant. If $s = 0$, $v = v_0$ when $t = 0$, determine the velocity of the particle as a function of time t .

12-10. Car A starts from rest at $t = 0$ and travels along a straight road with a constant acceleration of 6 ft/s^2 until it reaches a speed of 80 ft/s. Afterwards it maintains this speed. Also, when $t = 0$, car B located 6000 ft down the road is traveling towards A at a constant speed of 60 ft/s. Determine the distance traveled by car A when they pass each other.



Prob. 12-10

12-11. A particle travels along a straight line with a velocity $v = (12 - 3t^2) \text{ m/s}$, where t is in seconds. When $t = 1 \text{ s}$, the particle is located 10 m to the left of the origin. Determine the acceleration when $t = 4 \text{ s}$, the displacement from $t = 0$ to $t = 10 \text{ s}$, and the distance the particle travels during this time period.

*12-12. A sphere is fired downwards into a medium with an initial speed of 27 m/s. If it experiences a deceleration of $a = (-6t) \text{ m/s}^2$, where t is in seconds, determine the distance traveled before it stops.

•12-13. A particle travels along a straight line such that in 2 s it moves from an initial position $s_A = +0.5 \text{ m}$ to a position $s_B = -1.5 \text{ m}$. Then in another 4 s it moves from s_B to $s_C = +2.5 \text{ m}$. Determine the particle's average velocity and average speed during the 6-s time interval.

12-14. A particle travels along a straight-line path such that in 4 s it moves from an initial position $s_A = -8 \text{ m}$ to a position $s_B = +3 \text{ m}$. Then in another 5 s it moves from s_B to $s_C = -6 \text{ m}$. Determine the particle's average velocity and average speed during the 9-s time interval.

12–15. Tests reveal that a normal driver takes about 0.75 s before he or she can *react* to a situation to avoid a collision. It takes about 3 s for a driver having 0.1 % alcohol in his system to do the same. If such drivers are traveling on a straight road at 30 mph (44 ft/s) and their cars can decelerate at 2 ft/s^2 , determine the shortest stopping distance d for each from the moment they see the pedestrians. *Moral:* If you must drink, please don't drive!



Prob. 12–15

***12–16.** As a train accelerates uniformly it passes successive kilometer marks while traveling at velocities of 2 m/s and then 10 m/s. Determine the train's velocity when it passes the next kilometer mark and the time it takes to travel the 2-km distance.

***12–17.** A ball is thrown with an upward velocity of 5 m/s from the top of a 10-m high building. One second later another ball is thrown vertically from the ground with a velocity of 10 m/s. Determine the height from the ground where the two balls pass each other.

12–18. A car starts from rest and moves with a constant acceleration of 1.5 m/s^2 until it achieves a velocity of 25 m/s. It then travels with constant velocity for 60 seconds. Determine the average speed and the total distance traveled.

12–19. A car is to be hoisted by elevator to the fourth floor of a parking garage, which is 48 ft above the ground. If the elevator can accelerate at 0.6 ft/s^2 , decelerate at 0.3 ft/s^2 , and reach a maximum speed of 8 ft/s, determine the shortest time to make the lift, starting from rest and ending at rest.

***12–20.** A particle is moving along a straight line such that its speed is defined as $v = (-4s^2) \text{ m/s}$, where s is in meters. If $s = 2 \text{ m}$ when $t = 0$, determine the velocity and acceleration as functions of time.

***12–21.** Two particles A and B start from rest at the origin $s = 0$ and move along a straight line such that $a_A = (6t - 3) \text{ ft/s}^2$ and $a_B = (12t^2 - 8) \text{ ft/s}^2$, where t is in seconds. Determine the distance between them when $t = 4 \text{ s}$ and the total distance each has traveled in $t = 4 \text{ s}$.

12–22. A particle moving along a straight line is subjected to a deceleration $a = (-2v^3) \text{ m/s}^2$, where v is in m/s. If it has a velocity $v = 8 \text{ m/s}$ and a position $s = 10 \text{ m}$ when $t = 0$, determine its velocity and position when $t = 4 \text{ s}$.

12–23. A particle is moving along a straight line such that its acceleration is defined as $a = (-2v) \text{ m/s}^2$, where v is in meters per second. If $v = 20 \text{ m/s}$ when $s = 0$ and $t = 0$, determine the particle's position, velocity, and acceleration as functions of time.

***12–24.** A particle starts from rest and travels along a straight line with an acceleration $a = (30 - 0.2v) \text{ ft/s}^2$, where v is in ft/s. Determine the time when the velocity of the particle is $v = 30 \text{ ft/s}$.

***12–25.** When a particle is projected vertically upwards with an initial velocity of v_0 , it experiences an acceleration $a = -(g + kv^2)$, where g is the acceleration due to gravity, k is a constant and v is the velocity of the particle. Determine the maximum height reached by the particle.

12–26. The acceleration of a particle traveling along a straight line is $a = (0.02e^t) \text{ m/s}^2$, where t is in seconds. If $v = 0$, $s = 0$ when $t = 0$, determine the velocity and acceleration of the particle at $s = 4 \text{ m}$.

12–27. A particle moves along a straight line with an acceleration of $a = 5/(3s^{1/3} + s^{5/2}) \text{ m/s}^2$, where s is in meters. Determine the particle's velocity when $s = 2 \text{ m}$, if it starts from rest when $s = 1 \text{ m}$. Use Simpson's rule to evaluate the integral.

***12–28.** If the effects of atmospheric resistance are accounted for, a falling body has an acceleration defined by the equation $a = 9.81[1 - v^2(10^{-4})] \text{ m/s}^2$, where v is in m/s and the positive direction is downward. If the body is released from rest at a very *high altitude*, determine (a) the velocity when $t = 5 \text{ s}$, and (b) the body's terminal or maximum attainable velocity (as $t \rightarrow \infty$).

12-29. The position of a particle along a straight line is given by $s = (1.5t^3 - 13.5t^2 + 22.5t)$ ft, where t is in seconds. Determine the position of the particle when $t = 6$ s and the total distance it travels during the 6-s time interval. *Hint:* Plot the path to determine the total distance traveled.

12-30. The velocity of a particle traveling along a straight line is $v = v_0 - ks$, where k is constant. If $s = 0$ when $t = 0$, determine the position and acceleration of the particle as a function of time.

12-31. The acceleration of a particle as it moves along a straight line is given by $a = (2t - 1)$ m/s², where t is in seconds. If $s = 1$ m and $v = 2$ m/s when $t = 0$, determine the particle's velocity and position when $t = 6$ s. Also, determine the total distance the particle travels during this time period.

12-32. Ball A is thrown vertically upward from the top of a 30-m-high-building with an initial velocity of 5 m/s. At the same instant another ball B is thrown upward from the ground with an initial velocity of 20 m/s. Determine the height from the ground and the time at which they pass.

12-33. A motorcycle starts from rest at $t = 0$ and travels along a straight road with a constant acceleration of 6 ft/s² until it reaches a speed of 50 ft/s. Afterwards it maintains this speed. Also, when $t = 0$, a car located 6000 ft down the road is traveling toward the motorcycle at a constant speed of 30 ft/s. Determine the time and the distance traveled by the motorcycle when they pass each other.

12-34. A particle moves along a straight line with a velocity $v = (200s)$ mm/s, where s is in millimeters. Determine the acceleration of the particle at $s = 2000$ mm. How long does the particle take to reach this position if $s = 500$ mm when $t = 0$?

12-35. A particle has an initial speed of 27 m/s. If it experiences a deceleration of $a = (-6t)$ m/s², where t is in seconds, determine its velocity, after it has traveled 10 m. How much time does this take?

12-36. The acceleration of a particle traveling along a straight line is $a = (8 - 2s)$ m/s², where s is in meters. If $v = 0$ at $s = 0$, determine the velocity of the particle at $s = 2$ m, and the position of the particle when the velocity is maximum.

12-37. Ball A is thrown vertically upwards with a velocity of v_0 . Ball B is thrown upwards from the same point with the same velocity t seconds later. Determine the elapsed time $t < 2v_0/g$ from the instant ball A is thrown to when the balls pass each other, and find the velocity of each ball at this instant.

12-38. As a body is projected to a high altitude above the earth's surface, the variation of the acceleration of gravity with respect to altitude y must be taken into account. Neglecting air resistance, this acceleration is determined from the formula $a = -g_0[R^2/(R + y)^2]$, where g_0 is the constant gravitational acceleration at sea level, R is the radius of the earth, and the positive direction is measured upward. If $g_0 = 9.81$ m/s² and $R = 6356$ km, determine the minimum initial velocity (escape velocity) at which a projectile should be shot vertically from the earth's surface so that it does not fall back to the earth. *Hint:* This requires that $v = 0$ as $y \rightarrow \infty$.

12-39. Accounting for the variation of gravitational acceleration a with respect to altitude y (see Prob. 12-38), derive an equation that relates the velocity of a freely falling particle to its altitude. Assume that the particle is released from rest at an altitude y_0 from the earth's surface. With what velocity does the particle strike the earth if it is released from rest at an altitude $y_0 = 500$ km? Use the numerical data in Prob. 12-38.

12-40. When a particle falls through the air, its initial acceleration $a = g$ diminishes until it is zero, and thereafter it falls at a constant or terminal velocity v_f . If this variation of the acceleration can be expressed as $a = (g/v_f^2)(v_f^2 - v^2)$, determine the time needed for the velocity to become $v = v_f/2$. Initially the particle falls from rest.

12-41. A particle is moving along a straight line such that its position from a fixed point is $s = (12 - 15t^2 + 5t^3)$ m, where t is in seconds. Determine the total distance traveled by the particle from $t = 1$ s to $t = 3$ s. Also, find the average speed of the particle during this time interval.