

12.9 Absolute Dependent Motion Analysis of Two Particles

In some types of problems the motion of one particle will *depend* on the corresponding motion of another particle. This dependency commonly occurs if the particles, here represented by blocks, are interconnected by inextensible cords which are wrapped around pulleys. For example, the movement of block *A* downward along the inclined plane in Fig. 12-36 will cause a corresponding movement of block *B* up the other incline. We can show this mathematically by first specifying the location of the blocks using *position coordinates* s_A and s_B . Note that each of the coordinate axes is (1) measured from a *fixed point* (*O*) or *fixed datum line*, (2) measured along each inclined plane *in the direction of motion* of each block, and (3) has a positive sense from *C* to *A* and *D* to *B*. If the total cord length is l_T , the two position coordinates are related by the equation

$$s_A + l_{CD} + s_B = l_T$$

Here l_{CD} is the length of the cord passing over arc *CD*. Taking the time derivative of this expression, realizing that l_{CD} and l_T remain constant, while s_A and s_B measure the segments of the cord that change in length. We have

$$\frac{ds_A}{dt} + \frac{ds_B}{dt} = 0 \quad \text{or} \quad v_B = -v_A$$

The negative sign indicates that when block *A* has a velocity downward, i.e., in the direction of positive s_A , it causes a corresponding upward velocity of block *B*; i.e., *B* moves in the negative s_B direction.

In a similar manner, time differentiation of the velocities yields the relation between the accelerations, i.e.,

$$a_B = -a_A$$

A more complicated example is shown in Fig. 12-37*a*. In this case, the position of block *A* is specified by s_A , and the position of the *end* of the cord from which block *B* is suspended is defined by s_B . As above, we have chosen position coordinates which (1) have their origin at fixed points or datums, (2) are measured in the direction of motion of each block, and (3) are positive to the right for s_A and positive downward for s_B . During the motion, the length of the red colored segments of the cord in Fig. 12-37*a* remains constant. If l represents the total length of cord minus these segments, then the position coordinates can be related by the equation

$$2s_B + h + s_A = l$$

Since l and h are constant during the motion, the two time derivatives yield

$$2v_B = -v_A \quad 2a_B = -a_A$$

Hence, when *B* moves downward ($+s_B$), *A* moves to the left ($-s_A$) with twice the motion.

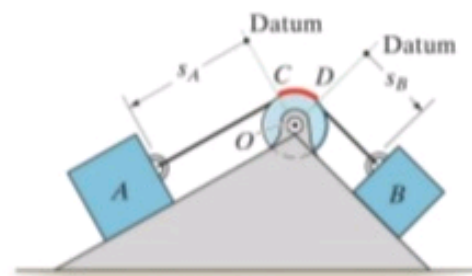


Fig. 12-36

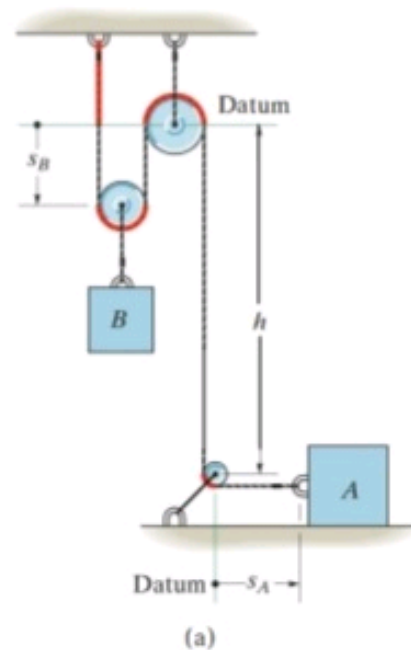


Fig. 12-37

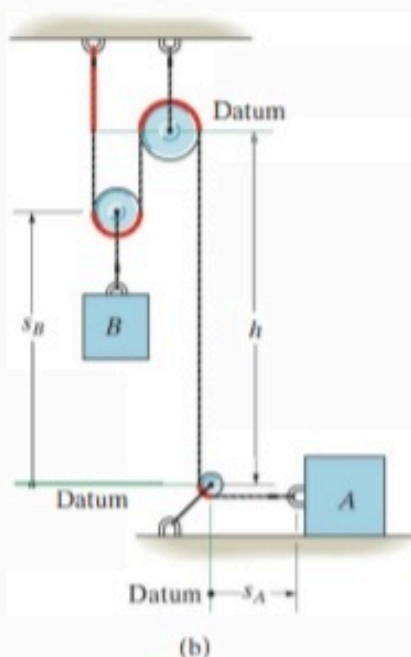


Fig. 12-37 (cont.)

This example can also be worked by defining the position of block B from the center of the bottom pulley (a fixed point), Fig. 12-37*b*. In this case

$$2(h - s_B) + h + s_A = l$$

Time differentiation yields

$$2v_B = v_A \quad 2a_B = a_A$$

Here the signs are the same. Why?

Procedure for Analysis

The above method of relating the dependent motion of one particle to that of another can be performed using algebraic scalars or position coordinates provided each particle moves along a rectilinear path. When this is the case, only the magnitudes of the velocity and acceleration of the particles will change, not their line of direction.

Position-Coordinate Equation.

- Establish each position coordinate with an origin located at a *fixed* point or datum.
- It is *not necessary* that the *origin* be the *same* for each of the coordinates; however, it is *important* that each coordinate axis selected be directed along the *path of motion* of the particle.
- Using geometry or trigonometry, relate the position coordinates to the total length of the cord, l_T , or to that portion of cord, l , which *excludes* the segments that do not change length as the particles move—such as arc segments wrapped over pulleys.
- If a problem involves a *system* of two or more cords wrapped around pulleys, then the position of a point on one cord must be related to the position of a point on another cord using the above procedure. Separate equations are written for a fixed length of each cord of the system and the positions of the two particles are then related by these equations (see Examples 12.22 and 12.23).

Time Derivatives.

- Two successive time derivatives of the position-coordinate equations yield the required velocity and acceleration equations which relate the motions of the particles.
- The signs of the terms in these equations will be consistent with those that specify the positive and negative sense of the position coordinates.



The motion of the traveling block on this oil rig depends upon the motion of the cable connected to the winch which operates it. It is important to be able to relate these motions in order to determine the power requirements of the winch and the force in the cable caused by any accelerated motion.

EXAMPLE 12.21

12

Determine the speed of block *A* in Fig. 12–38 if block *B* has an upward speed of 6 ft/s.

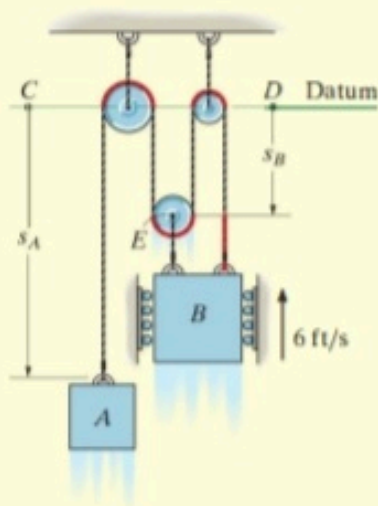


Fig. 12–38

SOLUTION

Position-Coordinate Equation. There is *one cord* in this system having segments which change length. Position coordinates s_A and s_B will be used since each is measured from a fixed point (*C* or *D*) and extends along each block's *path of motion*. In particular, s_B is directed to point *E* since motion of *B* and *E* is the *same*.

The red colored segments of the cord in Fig. 12–38 remain at a constant length and do not have to be considered as the blocks move. The remaining length of cord, l , is also constant and is related to the changing position coordinates s_A and s_B by the equation

$$s_A + 3s_B = l$$

Time Derivative. Taking the time derivative yields

$$v_A + 3v_B = 0$$

so that when $v_B = -6$ ft/s (upward),

$$v_A = 18 \text{ ft/s} \downarrow$$

Ans.

EXAMPLE 12.22

Determine the speed of A in Fig. 12–39 if B has an upward speed of 6 ft/s.

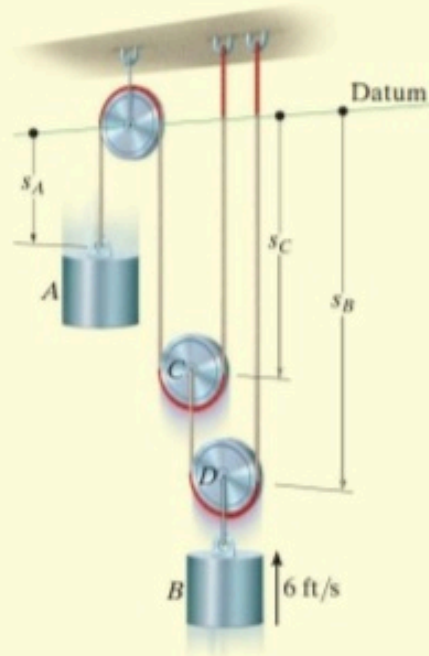


Fig. 12–39

SOLUTION

Position-Coordinate Equation. As shown, the positions of blocks A and B are defined using coordinates s_A and s_B . Since the system has *two cords* with segments that change length, it will be necessary to use a third coordinate, s_C , in order to relate s_A to s_B . In other words, the length of one of the cords can be expressed in terms of s_A and s_C , and the length of the other cord can be expressed in terms of s_B and s_C .

The red colored segments of the cords in Fig. 12–39 do not have to be considered in the analysis. Why? For the remaining cord lengths, say l_1 and l_2 , we have

$$s_A + 2s_C = l_1 \quad s_B + (s_B - s_C) = l_2$$

Time Derivative. Taking the time derivative of these equations yields

$$v_A + 2v_C = 0 \quad 2v_B - v_C = 0$$

Eliminating v_C produces the relationship between the motions of each cylinder.

$$v_A + 4v_B = 0$$

so that when $v_B = -6$ ft/s (upward),

$$v_A = +24 \text{ ft/s} = 24 \text{ ft/s} \downarrow$$

Ans.

EXAMPLE 12.23

Determine the speed of block B in Fig. 12–40 if the end of the cord at A is pulled down with a speed of 2 m/s.

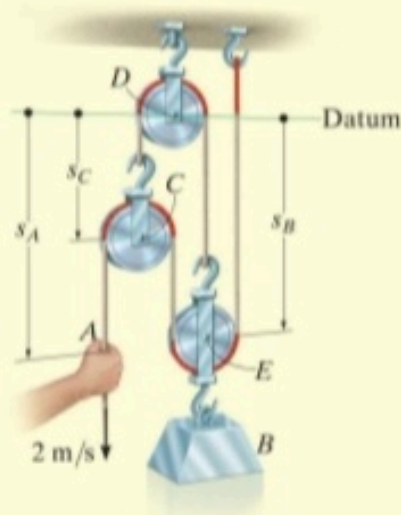


Fig. 12–40

SOLUTION

Position-Coordinate Equation. The position of point A is defined by s_A , and the position of block B is specified by s_B since point E on the pulley will have the *same motion* as the block. Both coordinates are measured from a horizontal datum passing through the *fixed* pin at pulley D . Since the system consists of *two* cords, the coordinates s_A and s_B cannot be related directly. Instead, by establishing a third position coordinate, s_C , we can now express the length of one of the cords in terms of s_B and s_C , and the length of the other cord in terms of s_A , s_B , and s_C .

Excluding the red colored segments of the cords in Fig. 12–40, the remaining constant cord lengths l_1 and l_2 (along with the hook and link dimensions) can be expressed as

$$\begin{aligned}s_C + s_B &= l_1 \\(s_A - s_C) + (s_B - s_C) + s_B &= l_2\end{aligned}$$

Time Derivative. The time derivative of each equation gives

$$\begin{aligned}v_C + v_B &= 0 \\v_A - 2v_C + 2v_B &= 0\end{aligned}$$

Eliminating v_C , we obtain

$$v_A + 4v_B = 0$$

so that when $v_A = 2$ m/s (downward),

$$v_B = -0.5 \text{ m/s} = 0.5 \text{ m/s} \uparrow \quad \text{Ans.}$$

EXAMPLE 12.24

A man at A is hoisting a safe S as shown in Fig. 12–41 by walking to the right with a constant velocity $v_A = 0.5$ m/s. Determine the velocity and acceleration of the safe when it reaches the elevation of 10 m. The rope is 30 m long and passes over a small pulley at D .

SOLUTION

Position-Coordinate Equation. This problem is unlike the previous examples since rope segment DA changes *both direction and magnitude*. However, the ends of the rope, which define the positions of S and A , are specified by means of the x and y coordinates since they must be measured from a fixed point and *directed along the paths of motion* of the ends of the rope.

The x and y coordinates may be related since the rope has a fixed length $l = 30$ m, which at all times is equal to the length of segment DA plus CD . Using the Pythagorean theorem to determine l_{DA} , we have

$$l_{DA} = \sqrt{(15)^2 + x^2}; \text{ also, } l_{CD} = 15 - y. \text{ Hence,}$$

$$l = l_{DA} + l_{CD}$$

$$30 = \sqrt{(15)^2 + x^2} + (15 - y)$$

$$y = \sqrt{225 + x^2} - 15 \quad (1)$$

Time Derivatives. Taking the time derivative, using the chain rule (see Appendix C), where $v_S = dy/dt$ and $v_A = dx/dt$, yields

$$\begin{aligned} v_S = \frac{dy}{dt} &= \left[\frac{1}{2} \frac{2x}{\sqrt{225 + x^2}} \right] \frac{dx}{dt} \\ &= \frac{x}{\sqrt{225 + x^2}} v_A \end{aligned} \quad (2)$$

At $y = 10$ m, x is determined from Eq. 1, i.e., $x = 20$ m. Hence, from Eq. 2 with $v_A = 0.5$ m/s,

$$v_S = \frac{20}{\sqrt{225 + (20)^2}} (0.5) = 0.4 \text{ m/s} = 400 \text{ mm/s} \uparrow \text{ Ans.}$$

The acceleration is determined by taking the time derivative of Eq. 2. Since v_A is constant, then $a_A = dv_A/dt = 0$, and we have

$$a_S = \frac{d^2 y}{dt^2} = \left[\frac{-x(dx/dt)}{(225 + x^2)^{3/2}} \right] x v_A + \left[\frac{1}{\sqrt{225 + x^2}} \right] \left(\frac{dx}{dt} \right) v_A + \left[\frac{1}{\sqrt{225 + x^2}} \right] x \frac{dv_A}{dt} = \frac{225 v_A^2}{(225 + x^2)^{3/2}}$$

At $x = 20$ m, with $v_A = 0.5$ m/s, the acceleration becomes

$$a_S = \frac{225(0.5 \text{ m/s})^2}{[225 + (20 \text{ m})^2]^{3/2}} = 0.00360 \text{ m/s}^2 = 3.60 \text{ mm/s}^2 \uparrow \text{ Ans.}$$

NOTE: The constant velocity at A causes the other end C of the rope to have an acceleration since v_A causes segment DA to change its direction as well as its length.

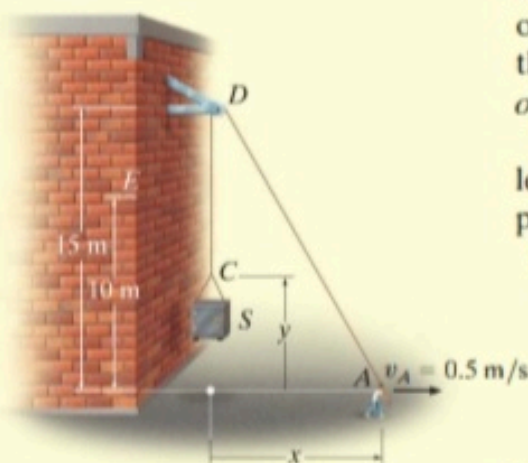
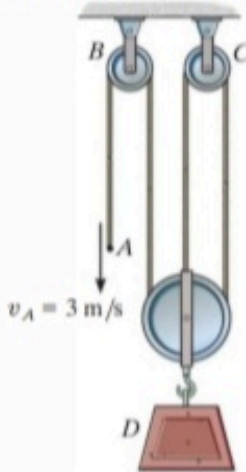


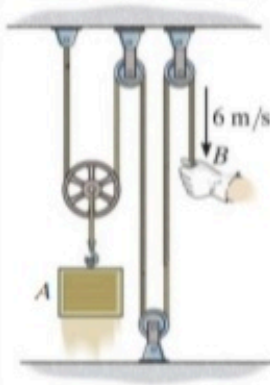
Fig. 12–41

F12-39. Determine the speed of block D if end A of the rope is pulled down with a speed of $v_A = 3 \text{ m/s}$.



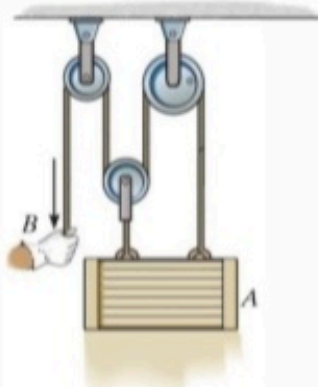
F12-39

F12-40. Determine the speed of block A if end B of the rope is pulled down with a speed of 6 m/s .



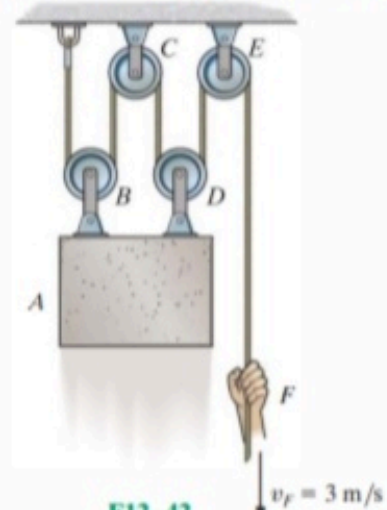
F12-40

F12-41. Determine the speed of block A if end B of the rope is pulled down with a speed of 1.5 m/s .



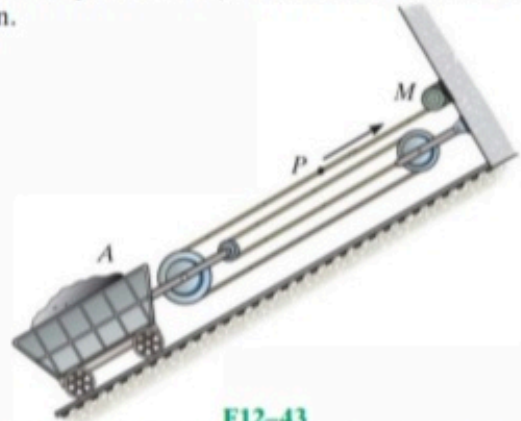
F12-41

F12-42. Determine the speed of block A if end F of the rope is pulled down with a speed of $v_F = 3 \text{ m/s}$.



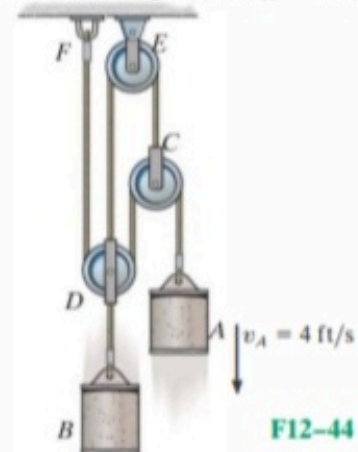
F12-42

F12-43. Determine the speed of car A if point P on the cable has a speed of 4 m/s when the motor M winds the cable in.



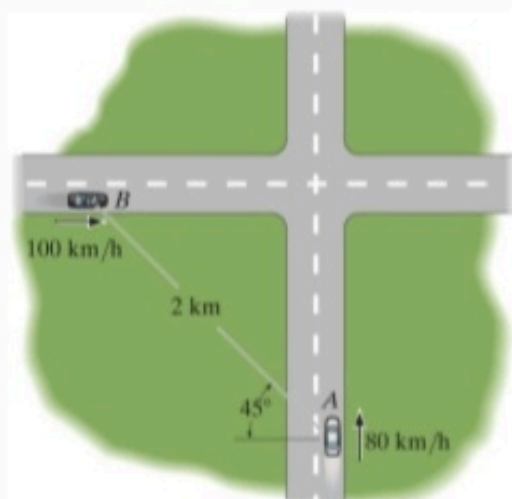
F12-43

F12-44. Determine the speed of cylinder B if cylinder A moves downward with a speed of $v_A = 4 \text{ ft/s}$.



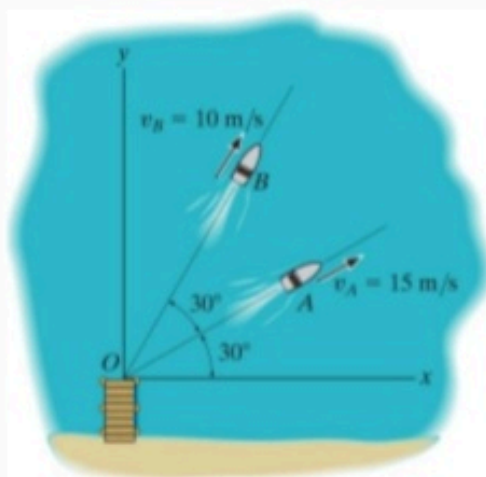
F12-44

F12-45. Car A is traveling with a constant speed of 80 km/h due north, while car B is traveling with a constant speed of 100 km/h due east. Determine the velocity of car B relative to car A .



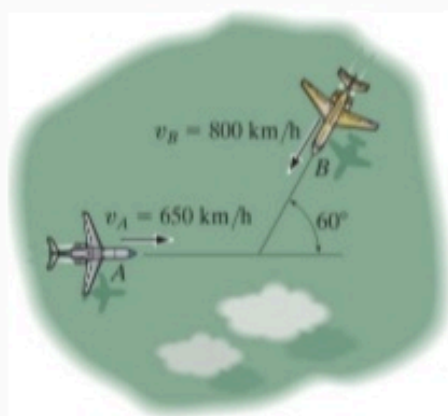
F12-45

F12-47. The boats A and B travel with constant speeds of $v_A = 15$ m/s and $v_B = 10$ m/s when they leave the pier at O at the same time. Determine the distance between them when $t = 4$ s.



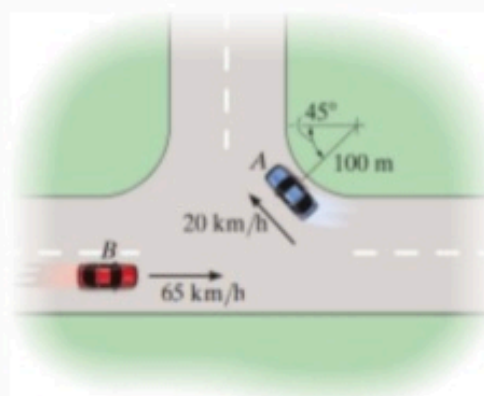
F12-47

F12-46. Two planes A and B are traveling with the constant velocities shown. Determine the magnitude and direction of the velocity of plane B relative to plane A .



F12-46

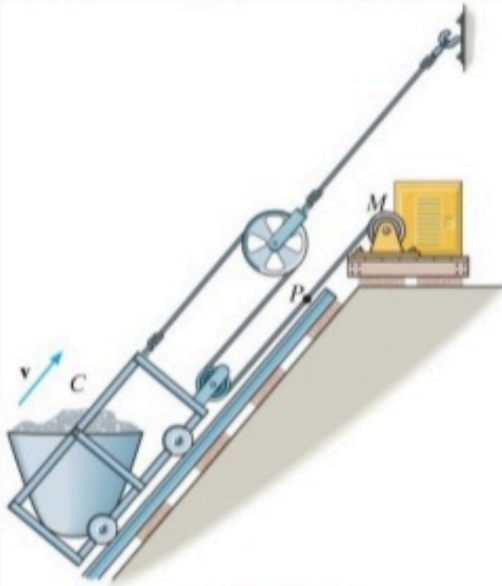
F12-48. At the instant shown, cars A and B are traveling at the speeds shown. If B is accelerating at 1200 km/h² while A maintains a constant speed, determine the velocity and acceleration of A with respect to B .



F12-48

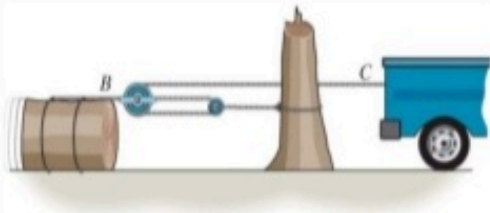
PROBLEMS

12–195. The mine car C is being pulled up the incline using the motor M and the rope-and-pulley arrangement shown. Determine the speed v_P at which a point P on the cable must be traveling toward the motor to move the car up the plane with a constant speed of $v = 2$ m/s.



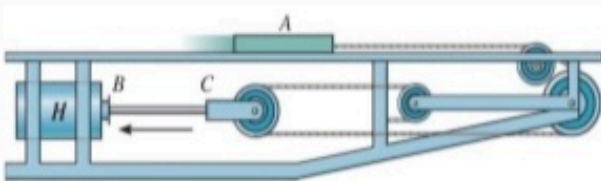
Prob. 12–195

***12–196.** Determine the displacement of the log if the truck at C pulls the cable 4 ft to the right.



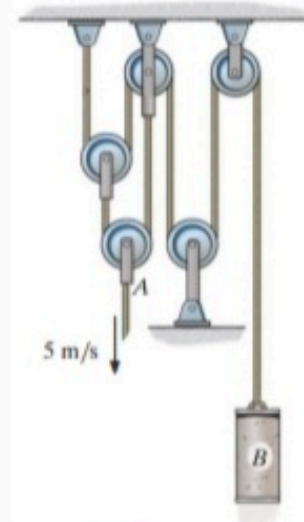
Prob. 12–196

•12–197. If the hydraulic cylinder H draws in rod BC at 2 ft/s, determine the speed of slider A .



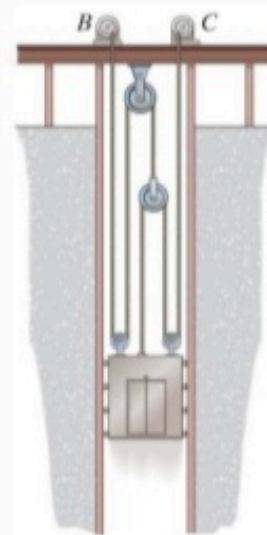
Prob. 12–197

12–198. If end A of the rope moves downward with a speed of 5 m/s, determine the speed of cylinder B .



Prob. 12–198

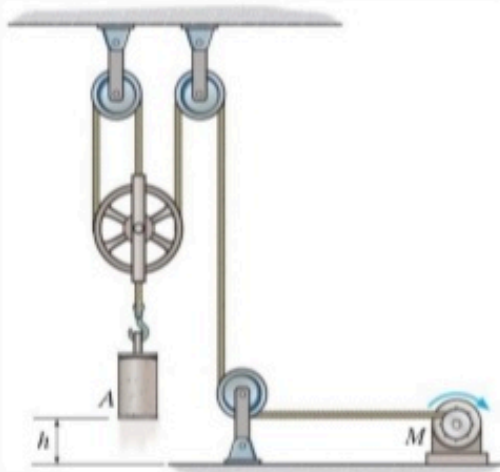
12–199. Determine the speed of the elevator if each motor draws in the cable with a constant speed of 5 m/s.



Prob. 12–199

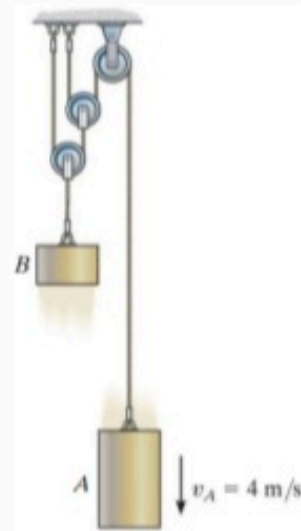
***12-200.** Determine the speed of cylinder A , if the rope is drawn towards the motor M at a constant rate of 10 m/s.

***12-201.** If the rope is drawn towards the motor M at a speed of $v_M = (5t^{3/2})$ m/s, where t is in seconds, determine the speed of cylinder A when $t = 1$ s.



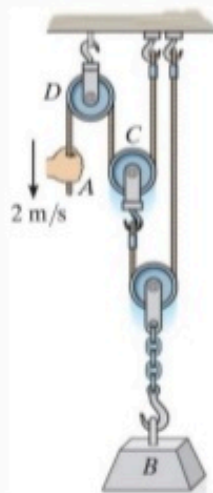
Probs. 12-200/201

12-203. Determine the speed of B if A is moving downwards with a speed of $v_A = 4$ m/s at the instant shown.



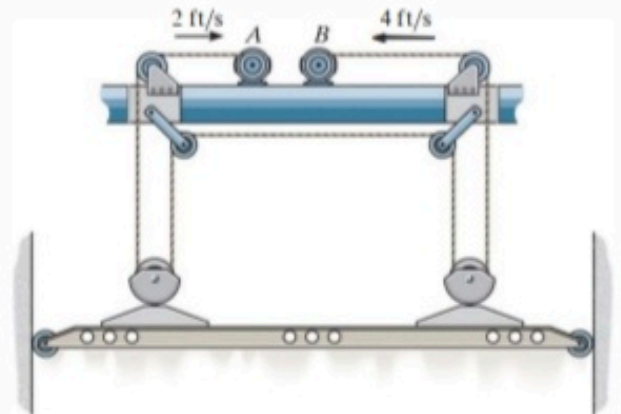
Prob. 12-203

12-202. If the end of the cable at A is pulled down with a speed of 2 m/s, determine the speed at which block B rises.



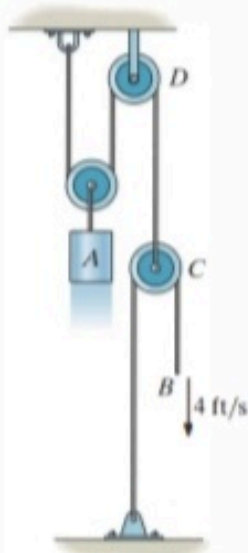
Prob. 12-202

***12-204.** The crane is used to hoist the load. If the motors at A and B are drawing in the cable at a speed of 2 ft/s and 4 ft/s, respectively, determine the speed of the load.



Prob. 12-204

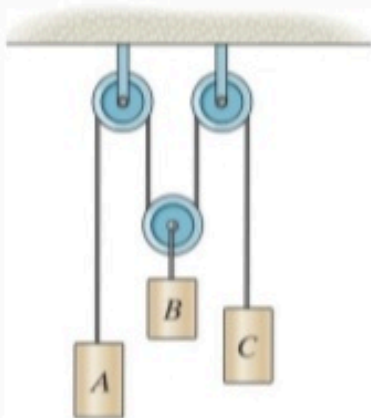
12 •12-205. The cable at B is pulled downwards at 4 ft/s , and the speed is decreasing at 2 ft/s^2 . Determine the velocity and acceleration of block A at this instant.



Prob. 12-205

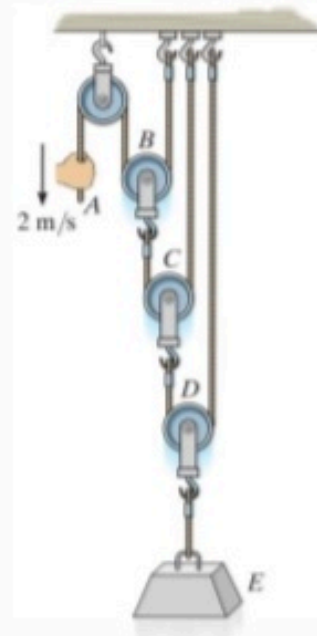
12-206. If block A is moving downward with a speed of 4 ft/s while C is moving up at 2 ft/s , determine the speed of block B .

12-207. If block A is moving downward at 6 ft/s while block C is moving down at 18 ft/s , determine the speed of block B .



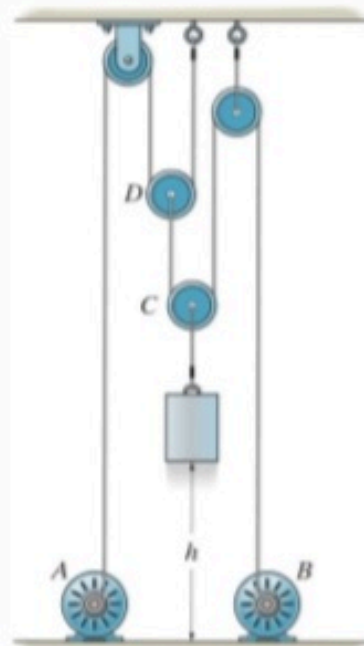
Probs. 12-206/207

*12-208. If the end of the cable at A is pulled down with a speed of 2 m/s , determine the speed at which block E rises.



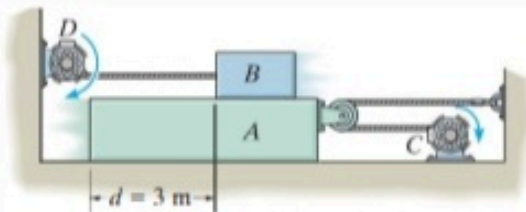
Prob. 12-208

•12-209. If motors at A and B draw in their attached cables with an acceleration of $a = (0.2t) \text{ m/s}^2$, where t is in seconds, determine the speed of the block when it reaches a height of $h = 4 \text{ m}$, starting from rest at $h = 0$. Also, how much time does it take to reach this height?



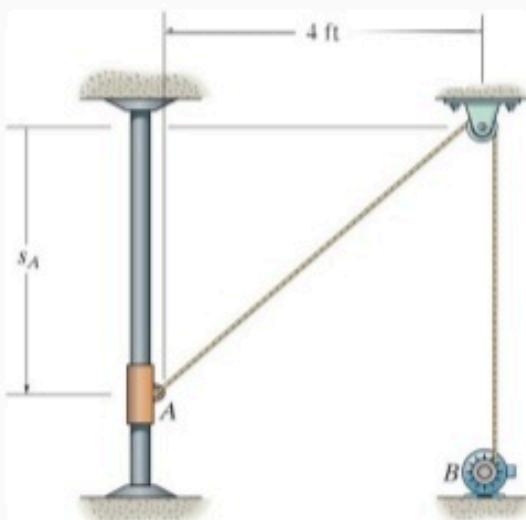
Prob. 12-209

12-210. The motor at C pulls in the cable with an acceleration $a_C = (3t^2) \text{ m/s}^2$, where t is in seconds. The motor at D draws in its cable at $a_D = 5 \text{ m/s}^2$. If both motors start at the same instant from rest when $d = 3 \text{ m}$, determine (a) the time needed for $d = 0$, and (b) the velocities of blocks A and B when this occurs.



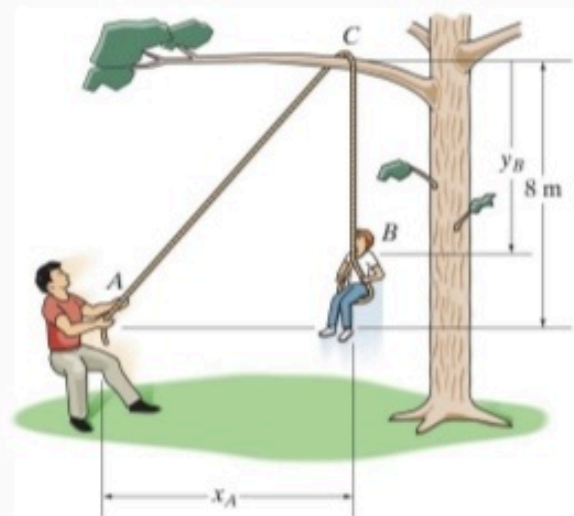
Prob. 12-210

12-211. The motion of the collar at A is controlled by a motor at B such that when the collar is at $s_A = 3 \text{ ft}$ it is moving upwards at 2 ft/s and decreasing at 1 ft/s^2 . Determine the velocity and acceleration of a point on the cable as it is drawn into the motor B at this instant.



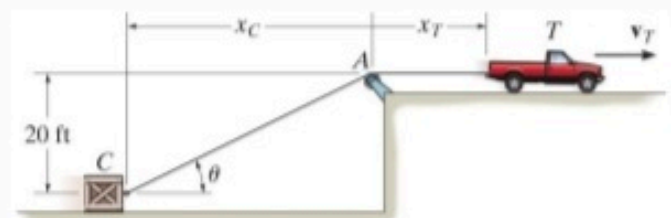
Prob. 12-211

***12-212.** The man pulls the boy up to the tree limb C by walking backward at a constant speed of 1.5 m/s . Determine the speed at which the boy is being lifted at the instant $x_A = 4 \text{ m}$. Neglect the size of the limb. When $x_A = 0$, $y_B = 8 \text{ m}$, so that A and B are coincident, i.e., the rope is 16 m long.



Probs. 12-212/213

12-214. If the truck travels at a constant speed of $v_T = 6 \text{ ft/s}$, determine the speed of the crate for any angle θ of the rope. The rope has a length of 100 ft and passes over a pulley of negligible size at A . *Hint:* Relate the coordinates x_T and x_C to the length of the rope and take the time derivative. Then substitute the trigonometric relation between x_C and θ .



Prob. 12-214

Absolute Dependent Motion of Two Particles

The dependent motion of blocks that are suspended from pulleys and cables can be related by the geometry of the system. This is done by first establishing position coordinates, measured from a fixed origin to each block. Each coordinate must be directed along the line of motion of a block.

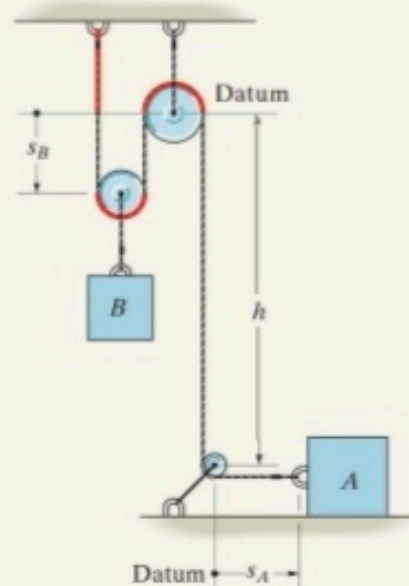
Using geometry and/or trigonometry, the coordinates are then related to the cable length in order to formulate a position coordinate equation.

The first time derivative of this equation gives a relationship between the velocities of the blocks, and a second time derivative gives the relation between their accelerations.

$$2s_B + h + s_A = l$$

$$2v_B = -v_A$$

$$2a_B = -a_A$$



Relative-Motion Analysis Using Translating Axes

If two particles A and B undergo independent motions, then these motions can be related to their relative motion using a *translating set of axes* attached to one of the particles (A).

For planar motion, each vector equation produces two scalar equations, one in the x , and the other in the y direction. For solution, the vectors can be expressed in Cartesian form, or the x and y scalar components can be written directly.

$$\mathbf{r}_B = \mathbf{r}_A + \mathbf{r}_{B/A}$$

$$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A}$$

$$\mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A}$$

