

Kinetics of a Particle: Force and Acceleration

13

CHAPTER OBJECTIVES

- To state Newton's Second Law of Motion and to define mass and weight.
- To analyze the accelerated motion of a particle using the equation of motion with different coordinate systems.
- To investigate central-force motion and apply it to problems in space mechanics.

13.1 Newton's Second Law of Motion

Kinetics is a branch of dynamics that deals with the relationship between the change in motion of a body and the forces that cause this change. The basis for kinetics is Newton's second law, which states that when an *unbalanced force* acts on a particle, the particle will *accelerate* in the direction of the force with a magnitude that is proportional to the force.

This law can be verified experimentally by applying a known unbalanced force $\mathbf{F}$ to a particle, and then measuring the acceleration $\mathbf{a}$. Since the force and acceleration are directly proportional, the constant of proportionality, m , may be determined from the ratio $m = F/a$. This positive scalar m is called the *mass* of the particle. Being constant during any acceleration, m provides a quantitative measure of the resistance of the particle to a change in its velocity, that is its inertia.

If the mass of the particle is m , Newton's second law of motion may be written in mathematical form as

$$\mathbf{F} = m\mathbf{a}$$

The above equation, which is referred to as the *equation of motion*, is one of the most important formulations in mechanics.* As previously stated, its validity is based solely on *experimental evidence*. In 1905, however, Albert Einstein developed the theory of relativity and placed limitations on the use of Newton's second law for describing general particle motion. Through experiments it was proven that *time* is not an absolute quantity as assumed by Newton; and as a result, the equation of motion fails to predict the exact behavior of a particle, especially when the particle's speed approaches the speed of light (0.3 Gm/s). Developments of the theory of quantum mechanics by Erwin Schrödinger and others indicate further that conclusions drawn from using this equation are also invalid when particles are the size of an atom and move close to one another. For the most part, however, these requirements regarding particle speed and size are not encountered in engineering problems, so their effects will not be considered in this book.

Newton's Law of Gravitational Attraction. Shortly after formulating his three laws of motion, Newton postulated a law governing the mutual attraction between any two particles. In mathematical form this law can be expressed as

$$F = G \frac{m_1 m_2}{r^2} \tag{13-1}$$

where

F = force of attraction between the two particles

G = universal constant of gravitation; according to experimental evidence $G = 66.73(10^{-12}) \text{ m}^3/(\text{kg} \cdot \text{s}^2)$

m_1, m_2 = mass of each of the two particles

r = distance between the centers of the two particles

*Since m is constant, we can also write $\mathbf{F} = d(m\mathbf{v})/dt$, where $m\mathbf{v}$ is the particle's linear momentum. Here the unbalanced force acting on the particle is proportional to the time rate of change of the particle's linear momentum.

In the case of a particle located at or near the surface of the earth, the only gravitational force having any sizable magnitude is that between the earth and the particle. This force is termed the “weight” and, for our purpose, it will be the only gravitational force considered.

From Eq. 13–1, we can develop a general expression for finding the weight W of a particle having a mass $m_1 = m$. Let $m_2 = M_e$ be the mass of the earth and r the distance between the earth's center and the particle. Then, if $g = GM_e/r^2$, we have

$$W = mg$$

By comparison with $F = ma$, we term g the acceleration due to gravity. For most engineering calculations g is a point on the surface of the earth at sea level, and at a latitude of 45° , which is considered the “standard location.” Here the values $g = 9.81 \text{ m/s}^2 = 32.2 \text{ ft/s}^2$ will be used for calculations.

In the SI system the mass of the body is specified in kilograms, and the weight must be calculated using the above equation, Fig. 13–1a. Thus,

$$W = mg \text{ (N)} \quad (g = 9.81 \text{ m/s}^2) \quad (13-2)$$

As a result, a body of mass 1 kg has a weight of 9.81 N; a 2-kg body weighs 19.62 N; and so on.

In the FPS system the weight of the body is specified in pounds. The mass is measured in slugs, a term derived from “sluggish” which refers to the body's inertia. It must be calculated, Fig. 13–1b, using

$$m = \frac{W}{g} \text{ (slug)} \quad (g = 32.2 \text{ ft/s}^2) \quad (13-3)$$

Therefore, a body weighing 32.2 lb has a mass of 1 slug; a 64.4-lb body has a mass of 2 slugs; and so on.

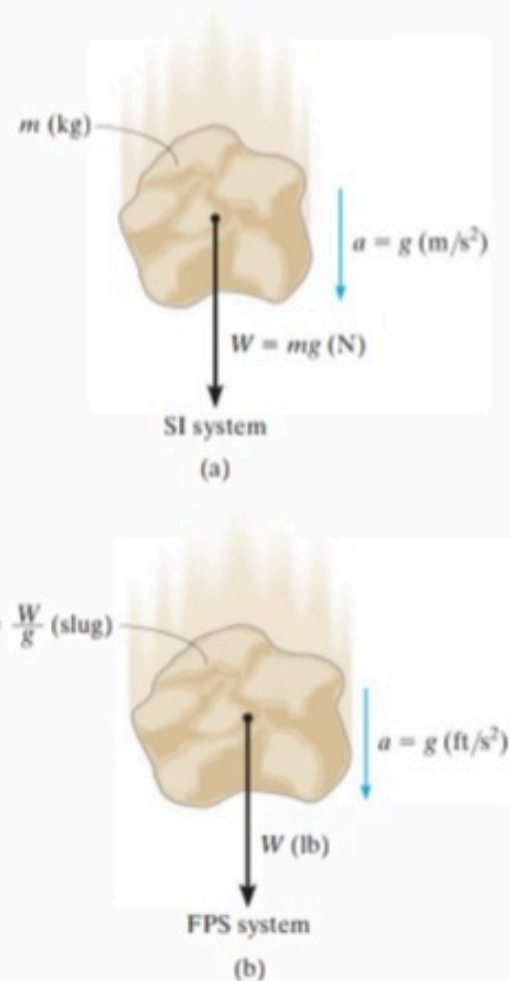


Fig. 13–1

13.2 The Equation of Motion

When more than one force acts on a particle, the resultant force is determined by a vector summation of all the forces; i.e., $\mathbf{F}_R = \Sigma \mathbf{F}$. For this more general case, the equation of motion may be written as

$$\Sigma \mathbf{F} = m\mathbf{a} \quad (13-4)$$

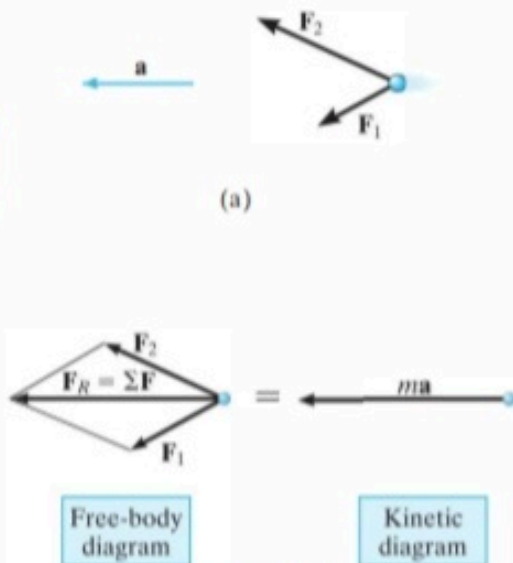


Fig. 13-2

To illustrate application of this equation, consider the particle shown in Fig. 13-2a, which has a mass m and is subjected to the action of two forces, $\mathbf{F}_1$ and $\mathbf{F}_2$. We can graphically account for the magnitude and direction of each force acting on the particle by drawing the particle's *free-body diagram*, Fig. 13-2b. Since the *resultant* of these forces *produces* the vector $m\mathbf{a}$, its magnitude and direction can be represented graphically on the *kinetic diagram*, shown in Fig. 13-2c.* The equal sign written between the diagrams symbolizes the *graphical* equivalency between the free-body diagram and the kinetic diagram; i.e., $\Sigma \mathbf{F} = m\mathbf{a}$.† In particular, note that if $\mathbf{F}_R = \Sigma \mathbf{F} = \mathbf{0}$, then the acceleration is also zero, so that the particle will either remain at *rest* or move along a straight-line path with *constant velocity*. Such are the conditions of *static equilibrium*, Newton's first law of motion.

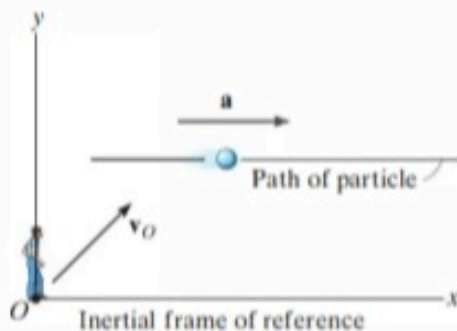


Fig. 13-3

Inertial Reference Frame. When applying the equation of motion, it is important that the acceleration of the particle be measured with respect to a reference frame that is *either fixed or translates with a constant velocity*. In this way, the observer will not accelerate and measurements of the particle's acceleration will be the *same* from *any* reference of this type. Such a frame of reference is commonly known as a *Newtonian or inertial reference frame*, Fig. 13-3.

When studying the motions of rockets and satellites, it is justifiable to consider the inertial reference frame as fixed to the stars, whereas dynamics problems concerned with motions on or near the surface of the earth may be solved by using an inertial frame which is assumed fixed to the earth. Even though the earth both rotates about its own axis and revolves about the sun, the accelerations created by these rotations are relatively small and so they can be neglected for most applications.

*Recall the free-body diagram considers the particle to be free of its surrounding supports and shows all the forces acting on the particle. The kinetic diagram pertains to the particle's motion as caused by the forces.

†The equation of motion can also be rewritten in the form $\Sigma \mathbf{F} - m\mathbf{a} = \mathbf{0}$. The vector $-m\mathbf{a}$ is referred to as the *inertia force vector*. If it is treated in the same way as a "force vector," then the state of "equilibrium" created is referred to as *dynamic equilibrium*. This method of application is often referred to as the *D'Alembert principle*, named after the French mathematician Jean le Rond d'Alembert.

13.3 Equation of Motion for a System of Particles

The equation of motion will now be extended to include a system of particles isolated within an enclosed region in space, as shown in Fig. 13-4a. In particular, there is no restriction in the way the particles are connected, so the following analysis applies equally well to the motion of a solid, liquid, or gas system.

At the instant considered, the arbitrary i -th particle, having a mass m_i , is subjected to a system of internal forces and a resultant external force. The *internal force*, represented symbolically as $\mathbf{f}_i$, is the resultant of all the forces the other particles exert on the i th particle. The *resultant external force* $\mathbf{F}_i$ represents, for example, the effect of gravitational, electrical, magnetic, or contact forces between the i th particle and adjacent bodies or particles *not* included within the system.

The free-body and kinetic diagrams for the i th particle are shown in Fig. 13-4b. Applying the equation of motion,

$$\Sigma \mathbf{F} = m\mathbf{a}; \quad \mathbf{F}_i + \mathbf{f}_i = m_i \mathbf{a}_i$$

When the equation of motion is applied to each of the other particles of the system, similar equations will result. And, if all these equations are added together *vectorially*, we obtain

$$\Sigma \mathbf{F}_i + \Sigma \mathbf{f}_i = \Sigma m_i \mathbf{a}_i$$

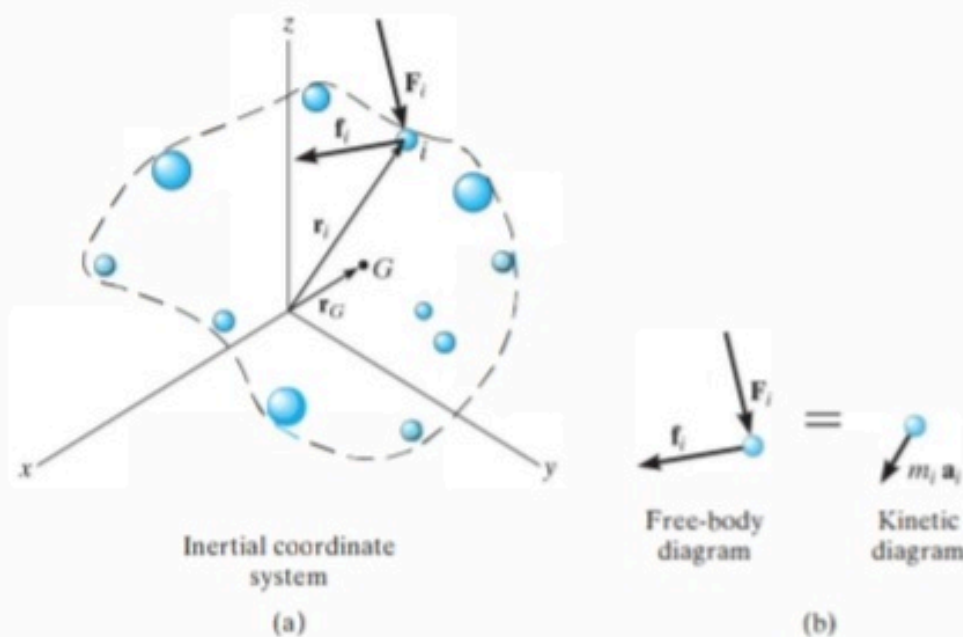


Fig. 13-4

The summation of the internal forces, if carried out, will equal zero, since internal forces between any two particles occur in equal but opposite collinear pairs. Consequently, only the sum of the external forces will remain, and therefore the equation of motion, written for the system of particles, becomes

$$\Sigma \mathbf{F}_i = \Sigma m_i \mathbf{a}_i \quad (13-5)$$

If $\mathbf{r}_G$ is a position vector which locates the *center of mass* G of the particles, Fig. 13-4a, then by definition of the center of mass, $m\mathbf{r}_G = \Sigma m_i \mathbf{r}_i$, where $m = \Sigma m_i$ is the total mass of all the particles. Differentiating this equation twice with respect to time, assuming that no mass is entering or leaving the system, yields

$$m\mathbf{a}_G = \Sigma m_i \mathbf{a}_i$$

Substituting this result into Eq. 13-5, we obtain

$$\Sigma \mathbf{F} = m\mathbf{a}_G \quad (13-6)$$

Hence, the sum of the external forces acting on the system of particles is equal to the total mass of the particles times the acceleration of its center of mass G . Since in reality all particles must have a finite size to possess mass, Eq. 13-6 justifies application of the equation of motion to a *body* that is represented as a single particle.

Important Points

- The equation of motion is based on experimental evidence and is valid only when applied within an inertial frame of reference.
- The equation of motion states that the *unbalanced force* on a particle causes it to *accelerate*.
- An inertial frame of reference does not rotate, rather its axes either translate with constant velocity or are at rest.
- Mass is a property of matter that provides a quantitative measure of its resistance to a change in velocity. It is an absolute quantity and so it does not change from one location to another.
- Weight is a force that is caused by the earth's gravitation. It is not absolute; rather it depends on the altitude of the mass from the earth's surface.

13.4 Equations of Motion: Rectangular Coordinates

When a particle moves relative to an inertial x, y, z frame of reference, the forces acting on the particle, as well as its acceleration, can be expressed in terms of their $\mathbf{i}, \mathbf{j}, \mathbf{k}$ components, Fig. 13–5. Applying the equation of motion, we have

$$\Sigma \mathbf{F} = m\mathbf{a}; \quad \Sigma F_x \mathbf{i} + \Sigma F_y \mathbf{j} + \Sigma F_z \mathbf{k} = m(a_x \mathbf{i} + a_y \mathbf{j} + a_z \mathbf{k})$$

For this equation to be satisfied, the respective $\mathbf{i}, \mathbf{j}, \mathbf{k}$ components on the left side must equal the corresponding components on the right side. Consequently, we may write the following three scalar equations:

$$\begin{aligned} \Sigma F_x &= ma_x \\ \Sigma F_y &= ma_y \\ \Sigma F_z &= ma_z \end{aligned} \quad (13-7)$$

In particular, if the particle is constrained to move only in the x – y plane, then the first two of these equations are used to specify the motion.

Fig. 13–5

Procedure for Analysis

The equations of motion are used to solve problems which require a relationship between the forces acting on a particle and the accelerated motion they cause.

Free-Body Diagram.

- Select the inertial coordinate system. Most often, rectangular or x, y, z coordinates are chosen to analyze problems for which the particle has *rectilinear motion*.
- Once the coordinates are established, draw the particle's free-body diagram. Drawing this diagram is *very important* since it provides a graphical representation that accounts for *all the forces* ($\Sigma \mathbf{F}$) which act on the particle, and thereby makes it possible to resolve these forces into their x, y, z components.
- The direction and sense of the particle's acceleration $\mathbf{a}$ should also be established. If the sense is unknown, for mathematical convenience assume that the sense of each acceleration component acts in the *same direction* as its *positive* inertial coordinate axis.
- The acceleration may be represented as the $m\mathbf{a}$ vector on the kinetic diagram.*
- Identify the unknowns in the problem.

*It is a convention in this text always to use the kinetic diagram as a graphical aid when developing the proofs and theory. The particle's acceleration or its components will be shown as blue colored vectors near the free-body diagram in the examples.

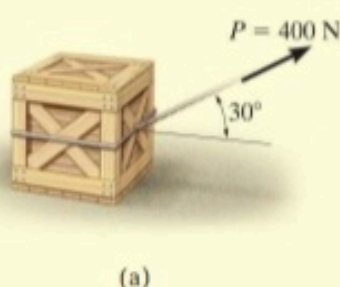
Equations of Motion.

- If the forces can be resolved directly from the free-body diagram, apply the equations of motion in their scalar component form.
- If the geometry of the problem appears complicated, which often occurs in three dimensions, Cartesian vector analysis can be used for the solution.
- *Friction.* If a moving particle contacts a rough surface, it may be necessary to use the *frictional equation*, which relates the frictional and normal forces $\mathbf{F}_f$ and $\mathbf{N}$ acting at the surface of contact by using the coefficient of kinetic friction, i.e., $F_f = \mu_k N$. Remember that $\mathbf{F}_f$ always acts on the free-body diagram such that it opposes the motion of the particle relative to the surface it contacts. If the particle is *on the verge* of relative motion, then the coefficient of static friction should be used.
- *Spring.* If the particle is connected to an *elastic spring* having negligible mass, the spring force F_s can be related to the deformation of the spring by the equation $F_s = ks$. Here k is the spring's stiffness measured as a force per unit length, and s is the stretch or compression defined as the difference between the deformed length l and the undeformed length l_0 , i.e., $s = l - l_0$.

Kinematics.

- If the velocity or position of the particle is to be found, it will be necessary to apply the necessary kinematic equations once the particle's acceleration is determined from $\Sigma \mathbf{F} = m\mathbf{a}$.
- If *acceleration* is a function of time, use $a = dv/dt$ and $v = ds/dt$ which, when integrated, yield the particle's velocity and position, respectively.
- If *acceleration* is a function of displacement, integrate $a ds = v dv$ to obtain the velocity as a function of position.
- If *acceleration is constant*, use $v = v_0 + a_c t$, $s = s_0 + v_0 t + \frac{1}{2} a_c t^2$, $v^2 = v_0^2 + 2a_c(s - s_0)$ to determine the velocity or position of the particle.
- If the problem involves the dependent motion of several particles, use the method outlined in Sec. 12.9 to relate their accelerations. In all cases, make sure the positive inertial coordinate directions used for writing the kinematic equations are the same as those used for writing the equations of motion; otherwise, simultaneous solution of the equations will result in errors.
- If the solution for an unknown vector component yields a negative scalar, it indicates that the component acts in the direction opposite to that which was assumed.

EXAMPLE 13.1



The 50-kg crate shown in Fig. 13-6a rests on a horizontal surface for which the coefficient of kinetic friction is $\mu_k = 0.3$. If the crate is subjected to a 400-N towing force as shown, determine the velocity of the crate in 3 s starting from rest.

SOLUTION

Using the equations of motion, we can relate the crate's acceleration to the force causing the motion. The crate's velocity can then be determined using kinematics.

Free-Body Diagram. The weight of the crate is $W = mg = 50 \text{ kg} (9.81 \text{ m/s}^2) = 490.5 \text{ N}$. As shown in Fig. 13-6b, the frictional force has a magnitude $F = \mu_k N_C$ and acts to the left, since it opposes the motion of the crate. The acceleration $\mathbf{a}$ is assumed to act horizontally, in the positive x direction. There are two unknowns, namely N_C and a .

Equations of Motion. Using the data shown on the free-body diagram, we have

$$\rightarrow \Sigma F_x = ma_x; \quad 400 \cos 30^\circ - 0.3 N_C = 50a \quad (1)$$

$$+\uparrow \Sigma F_y = ma_y; \quad N_C - 490.5 + 400 \sin 30^\circ = 0 \quad (2)$$

Solving Eq. 2 for N_C , substituting the result into Eq. 1, and solving for a yields

$$N_C = 290.5 \text{ N}$$

$$a = 5.185 \text{ m/s}^2$$

Kinematics. Notice that the acceleration is *constant*, since the applied force $\mathbf{P}$ is constant. Since the initial velocity is zero, the velocity of the crate in 3 s is

$$\begin{aligned} (\rightarrow) \quad v &= v_0 + a_c t = 0 + 5.185(3) \\ &= 15.6 \text{ m/s} \rightarrow \end{aligned}$$

Ans.

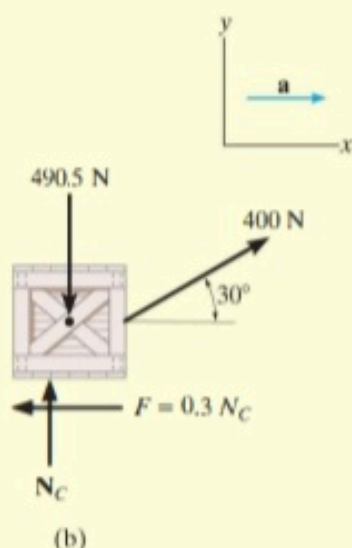
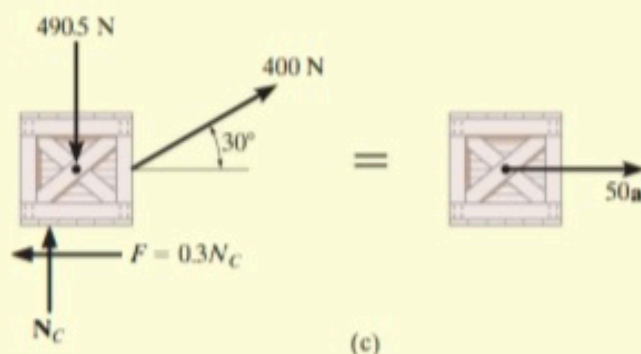


Fig. 13-6



NOTE: We can also use the alternative procedure of drawing the crate's free-body and kinetic diagrams, Fig. 13-6c, prior to applying the equations of motion.

EXAMPLE 13.2

A 10-kg projectile is fired vertically upward from the ground, with an initial velocity of 50 m/s, Fig. 13-7a. Determine the maximum height to which it will travel if (a) atmospheric resistance is neglected; and (b) atmospheric resistance is measured as $F_D = (0.01v^2)$ N, where v is the speed of the projectile at any instant, measured in m/s.

SOLUTION

In both cases the known force on the projectile can be related to its acceleration using the equation of motion. Kinematics can then be used to relate the projectile's acceleration to its position.

Part (a) Free-Body Diagram. As shown in Fig. 13-7b, the projectile's weight is $W = mg = 10(9.81) = 98.1$ N. We will assume the unknown acceleration a acts upward in the *positive* z direction.

Equation of Motion.

$$+\uparrow \Sigma F_z = ma_z; \quad -98.1 = 10a, \quad a = -9.81 \text{ m/s}^2$$

The result indicates that the projectile, like every object having free-flight motion near the earth's surface, is subjected to a *constant* downward acceleration of 9.81 m/s^2 .

Kinematics. Initially, $z_0 = 0$ and $v_0 = 50 \text{ m/s}$, and at the maximum height $z = h$, $v = 0$. Since the acceleration is *constant*, then

$$\begin{aligned} (+\uparrow) \quad v^2 &= v_0^2 + 2a_c(z - z_0) \\ 0 &= (50)^2 + 2(-9.81)(h - 0) \\ h &= 127 \text{ m} \end{aligned} \quad \text{Ans.}$$

Part (b) Free-Body Diagram. Since the force $F_D = (0.01v^2)$ N tends to retard the upward motion of the projectile, it acts downward as shown on the free-body diagram, Fig. 13-7c.

Equation of Motion.

$$+\uparrow \Sigma F_z = ma_z; \quad -0.01v^2 - 98.1 = 10a, \quad a = -(0.001v^2 + 9.81)$$

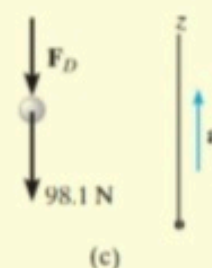
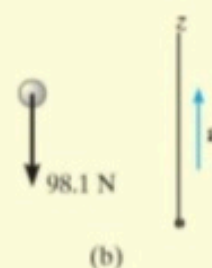
Kinematics. Here the acceleration is *not constant* since F_D depends on the velocity. Since $a = f(v)$, we can relate a to position using

$$(+\uparrow) a \, dz = v \, dv; \quad -(0.001v^2 + 9.81) \, dz = v \, dv$$

Separating the variables and integrating, realizing that initially $z_0 = 0$, $v_0 = 50 \text{ m/s}$ (positive upward), and at $z = h$, $v = 0$, we have

$$\begin{aligned} \int_0^h dz &= - \int_{50}^0 \frac{v \, dv}{0.001v^2 + 9.81} = -500 \ln(v^2 + 9810) \Big|_{50 \text{ m/s}}^0 \\ h &= 114 \text{ m} \end{aligned} \quad \text{Ans.}$$

NOTE: The answer indicates a lower elevation than that obtained in part (a) due to atmospheric resistance or drag.

**Fig. 13-7**

EXAMPLE 13.4

A smooth 2-kg collar C , shown in Fig. 13–9a, is attached to a spring having a stiffness $k = 3 \text{ N/m}$ and an unstretched length of 0.75 m . If the collar is released from rest at A , determine its acceleration and the normal force of the rod on the collar at the instant $y = 1 \text{ m}$.

SOLUTION

Free-Body Diagram. The free-body diagram of the collar when it is located at the arbitrary position y is shown in Fig. 13–9b. Furthermore, the collar is *assumed* to be accelerating so that “ a ” acts downward in the *positive* y direction. There are four unknowns, namely, N_C , F_s , a , and θ .

Equations of Motion.

$$\rightarrow \Sigma F_x = ma_x; \quad -N_C + F_s \cos \theta = 0 \quad (1)$$

$$+\downarrow \Sigma F_y = ma_y; \quad 19.62 - F_s \sin \theta = 2a \quad (2)$$

From Eq. 2 it is seen that the acceleration depends on the magnitude and direction of the spring force. Solution for N_C and a is possible once F_s and θ are known.

The magnitude of the spring force is a function of the stretch s of the spring; i.e., $F_s = ks$. Here the unstretched length is $AB = 0.75 \text{ m}$, Fig. 13–9a; therefore, $s = CB - AB = \sqrt{y^2 + (0.75)^2} - 0.75$. Since $k = 3 \text{ N/m}$, then

$$F_s = ks = 3(\sqrt{y^2 + (0.75)^2} - 0.75) \quad (3)$$

From Fig. 13–9a, the angle θ is related to y by trigonometry.

$$\tan \theta = \frac{y}{0.75} \quad (4)$$

Substituting $y = 1 \text{ m}$ into Eqs. 3 and 4 yields $F_s = 1.50 \text{ N}$ and $\theta = 53.1^\circ$. Substituting these results into Eqs. 1 and 2, we obtain

$$N_C = 0.900 \text{ N} \quad \text{Ans.}$$

$$a = 9.21 \text{ m/s}^2 \downarrow \quad \text{Ans.}$$

NOTE: This is not a case of constant acceleration, since the spring force changes both its magnitude and direction as the collar moves downward.

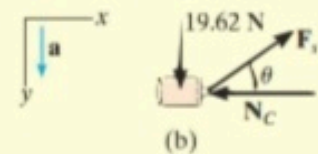
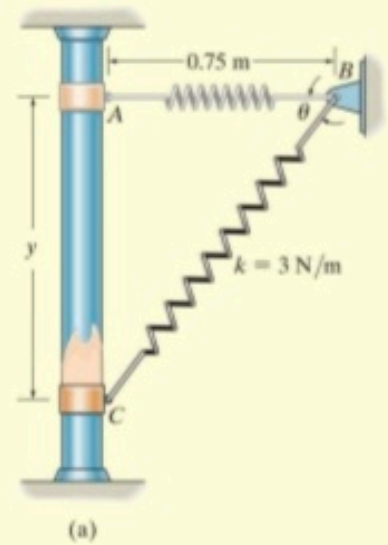
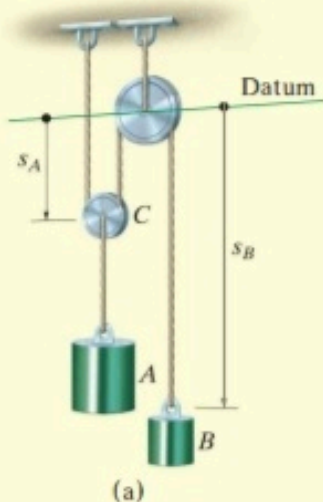


Fig. 13–9

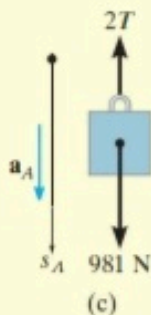
EXAMPLE 13.5



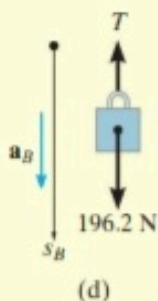
(a)



(b)



(c)



(d)

Fig. 13-10

The 100-kg block A shown in Fig. 13-10a is released from rest. If the masses of the pulleys and the cord are neglected, determine the speed of the 20-kg block B in 2 s.

SOLUTION

Free-Body Diagrams. Since the mass of the pulleys is *neglected*, then for pulley C , $ma = 0$ and we can apply $\Sigma F_y = 0$ as shown in Fig. 13-10b. The free-body diagrams for blocks A and B are shown in Fig. 13-10c and d, respectively. Notice that for A to remain stationary $T = 490.5$ N, whereas for B to remain static $T = 196.2$ N. Hence A will move down while B moves up. Although this is the case, we will assume both blocks accelerate downward, in the direction of $+s_A$ and $+s_B$. The three unknowns are T , a_A , and a_B .

Equations of Motion. Block A ,

$$+\downarrow \Sigma F_y = ma_y; \quad 981 - 2T = 100a_A \quad (1)$$

Block B ,

$$+\downarrow \Sigma F_y = ma_y; \quad 196.2 - T = 20a_B \quad (2)$$

Kinematics. The necessary third equation is obtained by relating a_A to a_B using a dependent motion analysis, discussed in Sect. 12.9. The coordinates s_A and s_B in Fig. 13-10a measure the positions of A and B from the fixed datum. It is seen that

$$2s_A + s_B = l$$

where l is constant and represents the total vertical length of cord. Differentiating this expression twice with respect to time yields

$$2a_A = -a_B \quad (3)$$

Notice that when writing Eqs. 1 to 3, the *positive direction was always assumed downward*. It is very important to be *consistent* in this assumption since we are seeking a simultaneous solution of equations. The results are

$$T = 327.0 \text{ N}$$

$$a_A = 3.27 \text{ m/s}^2$$

$$a_B = -6.54 \text{ m/s}^2$$

Hence when block A accelerates *downward*, block B accelerates *upward* as expected. Since a_B is constant, the velocity of block B in 2 s is thus

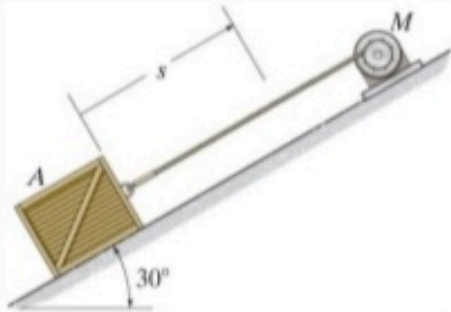
$$\begin{aligned} (+\downarrow) \quad v &= v_0 + a_B t \\ &= 0 + (-6.54)(2) \\ &= -13.1 \text{ m/s} \end{aligned}$$

Ans.

The negative sign indicates that block B is moving upward.

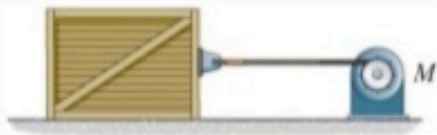
FUNDAMENTAL PROBLEMS

F13-1. The motor winds in the cable with a constant acceleration, such that the 20-kg crate moves a distance $s = 6$ m in 3 s, starting from rest. Determine the tension developed in the cable. The coefficient of kinetic friction between the crate and the plane is $\mu_k = 0.3$.



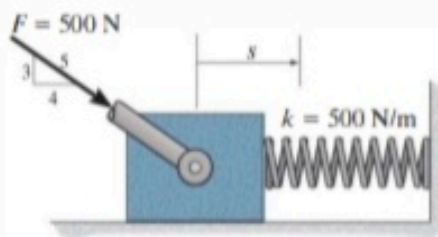
F13-1

F13-2. If motor M exerts a force of $F = (10t^2 + 100)$ N on the cable, where t is in seconds, determine the velocity of the 25-kg crate when $t = 4$ s. The coefficients of static and kinetic friction between the crate and the plane are $\mu_s = 0.3$ and $\mu_k = 0.25$, respectively. The crate is initially at rest.



F13-2

F13-3. A spring of stiffness $k = 500$ N/m is mounted against the 10-kg block. If the block is subjected to the force of $F = 500$ N, determine its velocity at $s = 0.5$ m. When $s = 0$, the block is at rest and the spring is uncompressed. The contact surface is smooth.



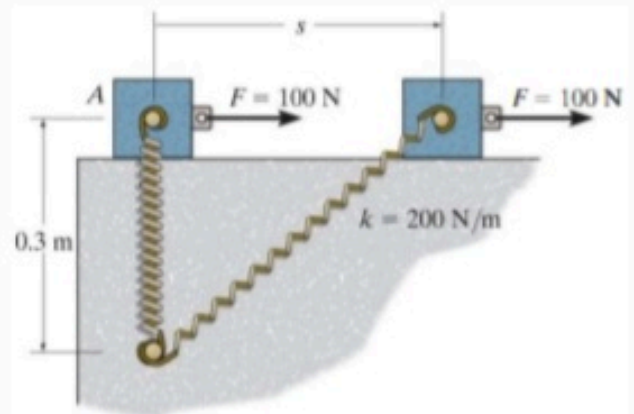
F13-3

F13-4. The 2-Mg car is being towed by a winch. If the winch exerts a force of $T = (100s)$ N on the cable, where s is the displacement of the car in meters, determine the speed of the car when $s = 10$ m, starting from rest. Neglect rolling resistance of the car.



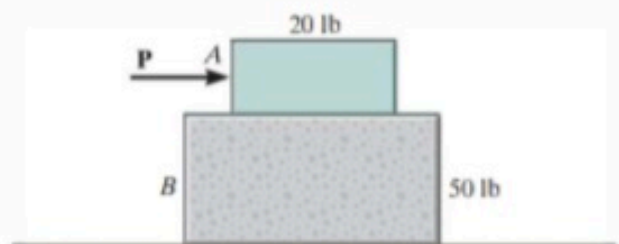
F13-4

F13-5. The spring has a stiffness $k = 200$ N/m and is unstretched when the 25-kg block is at A . Determine the acceleration of the block when $s = 0.4$ m. The contact surface between the block and the plane is smooth.



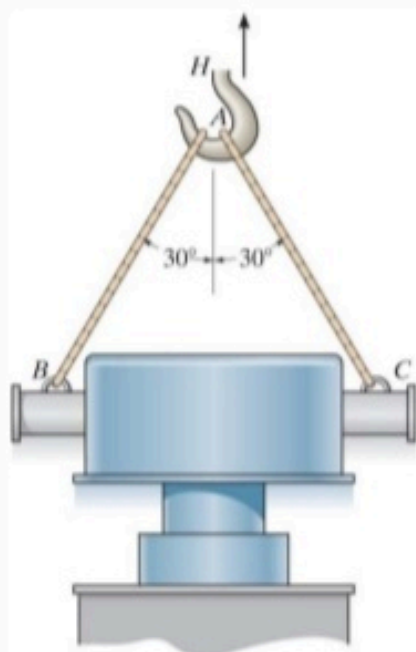
F13-5

F13-6. Block B rests upon a smooth surface. If the coefficients of static and kinetic friction between A and B are $\mu_s = 0.4$ and $\mu_k = 0.3$, respectively, determine the acceleration of each block if $P = 6$ lb.



F13-6

13-1. The casting has a mass of 3 Mg. Suspended in a vertical position and initially at rest, it is given an upward speed of 200 mm/s in 0.3 s using a crane hook H . Determine the tension in cables AC and AB during this time interval if the acceleration is constant.



Prob. 13-1

13-2. The 160-Mg train travels with a speed of 80 km/h when it starts to climb the slope. If the engine exerts a traction force F of $1/20$ of the weight of the train and the rolling resistance F_D is equal to $1/500$ of the weight of the train, determine the deceleration of the train.

13-3. The 160-Mg train starts from rest and begins to climb the slope as shown. If the engine exerts a traction force F of $1/8$ of the weight of the train, determine the speed of the train when it has traveled up the slope a distance of 1 km. Neglect rolling resistance.



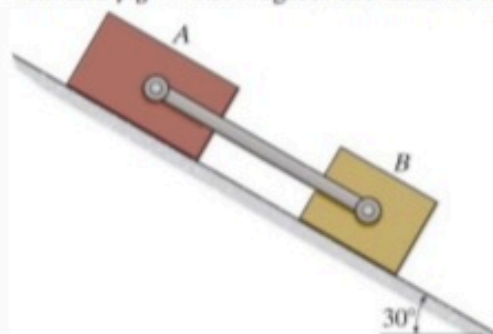
Probs. 13-2/3

13-4. The 2-Mg truck is traveling at 15 m/s when the brakes on all its wheels are applied, causing it to skid for a distance of 10 m before coming to rest. Determine the constant horizontal force developed in the coupling C , and the frictional force developed between the tires of the truck and the road during this time. The total mass of the boat and trailer is 1 Mg.



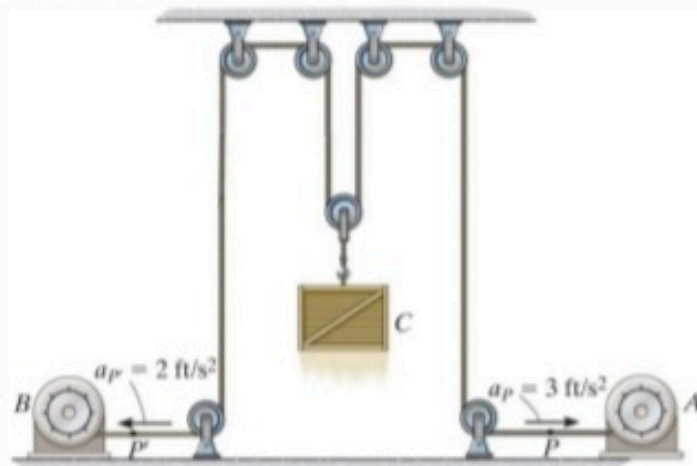
Prob. 13-4

13-5. If blocks A and B of mass 10 kg and 6 kg, respectively, are placed on the inclined plane and released, determine the force developed in the link. The coefficients of kinetic friction between the blocks and the inclined plane are $\mu_A = 0.1$ and $\mu_B = 0.3$. Neglect the mass of the link.



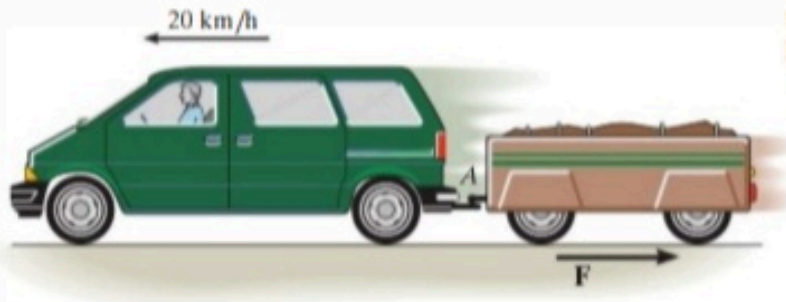
Prob. 13-5

13-6. Motors A and B draw in the cable with the accelerations shown. Determine the acceleration of the 300-lb crate C and the tension developed in the cable. Neglect the mass of all the pulleys.



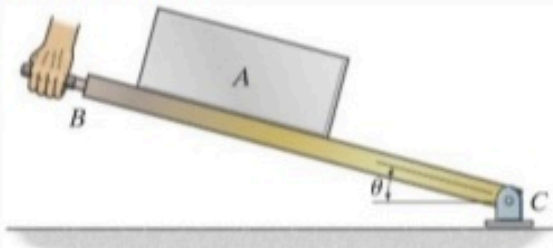
Prob. 13-6

13-7. The van is traveling at 20 km/h when the coupling of the trailer at *A* fails. If the trailer has a mass of 250 kg and coasts 45 m before coming to rest, determine the constant horizontal force *F* created by rolling friction which causes the trailer to stop.



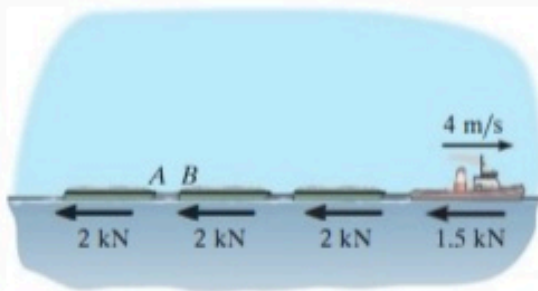
Prob. 13-7

***13-8.** If the 10-lb block *A* slides down the plane with a constant velocity when $\theta = 30^\circ$, determine the acceleration of the block when $\theta = 45^\circ$.



Prob. 13-8

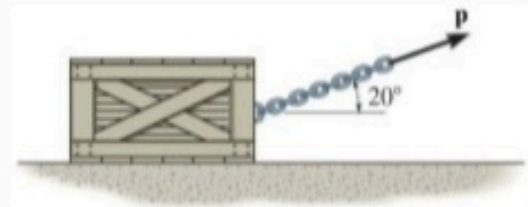
***13-9.** Each of the three barges has a mass of 30 Mg, whereas the tugboat has a mass of 12 Mg. As the barges are being pulled forward with a constant velocity of 4 m/s, the tugboat must overcome the frictional resistance of the water, which is 2 kN for each barge and 1.5 kN for the tugboat. If the cable between *A* and *B* breaks, determine the acceleration of the tugboat.



Prob. 13-9

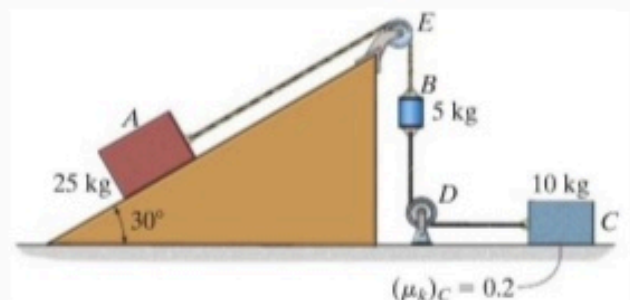
13-10. The crate has a mass of 80 kg and is being towed by a chain which is always directed at 20° from the horizontal as shown. If the magnitude of *P* is increased until the crate begins to slide, determine the crate's initial acceleration if the coefficient of static friction is $\mu_s = 0.5$ and the coefficient of kinetic friction is $\mu_k = 0.3$.

13-11. The crate has a mass of 80 kg and is being towed by a chain which is always directed at 20° from the horizontal as shown. Determine the crate's acceleration in $t = 2$ s if the coefficient of static friction is $\mu_s = 0.4$, the coefficient of kinetic friction is $\mu_k = 0.3$, and the towing force is $P = (90t^2)$ N, where t is in seconds.



Probs. 13-10/11

***13-12.** Determine the acceleration of the system and the tension in each cable. The inclined plane is smooth, and the coefficient of kinetic friction between the horizontal surface and block *C* is $(\mu_k)_C = 0.2$.



Prob. 13-12

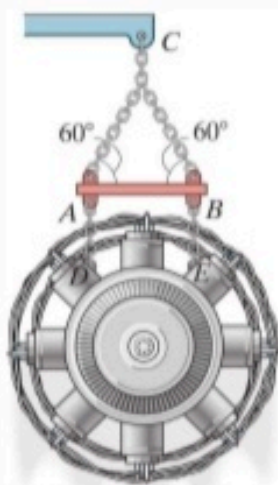
•13–13. The two boxcars *A* and *B* have a weight of 20 000 lb and 30 000 lb, respectively. If they coast freely down the incline when the brakes are applied to all the wheels of car *A* causing it to skid, determine the force in the coupling *C* between the two cars. The coefficient of kinetic friction between the wheels of *A* and the tracks is $\mu_k = 0.5$. The wheels of car *B* are free to roll. Neglect their mass in the calculation. *Suggestion:* Solve the problem by representing single resultant normal forces acting on *A* and *B*, respectively.



Prob. 13–13

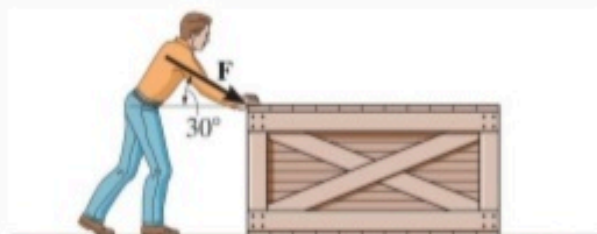
13–14. The 3.5-Mg engine is suspended from a spreader beam *AB* having a negligible mass and is hoisted by a crane which gives it an acceleration of 4 m/s^2 when it has a velocity of 2 m/s . Determine the force in chains *CA* and *CB* during the lift.

13–15. The 3.5-Mg engine is suspended from a spreader beam having a negligible mass and is hoisted by a crane which exerts a force of 40 kN on the hoisting cable. Determine the distance the engine is hoisted in 4 s, starting from rest.



Probs. 13–14/15

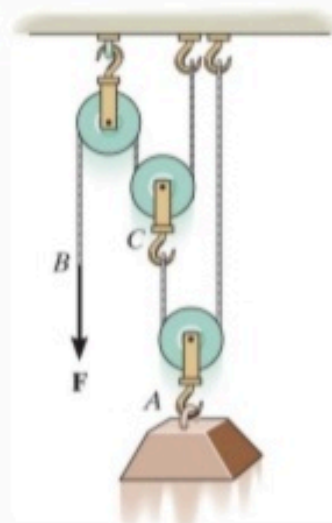
***13–16.** The man pushes on the 60-lb crate with a force *F*. The force is always directed down at 30° from horizontal as shown, and its magnitude is increased until the crate begins to slide. Determine the crate's initial acceleration if the coefficient of static friction is $\mu_s = 0.4$ and the coefficient of kinetic friction is $\mu_k = 0.3$.



Prob. 13–16

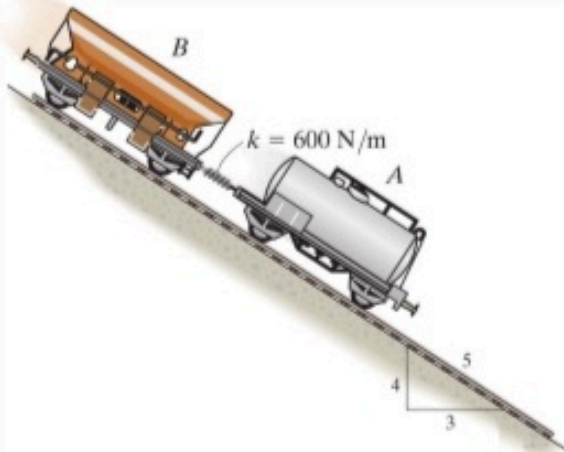
•13–17. A force of $F = 15 \text{ lb}$ is applied to the cord in order to lift the 30-lb block *A*. Determine how high the 30-lb block *A* rises in 2 s starting from rest. Neglect the weight of the pulleys and cord.

13–18. Determine the constant force *F* which must be applied to the cord in order to cause the 30-lb block *A* to have a speed of 12 ft/s when it has been displaced 4 ft upward starting from rest. Neglect the weight of the pulleys and cord.



Probs. 13–17/18

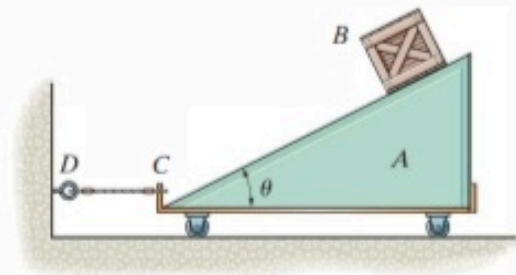
13–19. The 800-kg car at B is connected to the 350-kg car at A by a spring coupling. Determine the stretch in the spring if (a) the wheels of both cars are free to roll and (b) the brakes are applied to all four wheels of car B , causing the wheels to skid. Take $(\mu_k)_B = 0.4$. Neglect the mass of the wheels.



Prob. 13–19

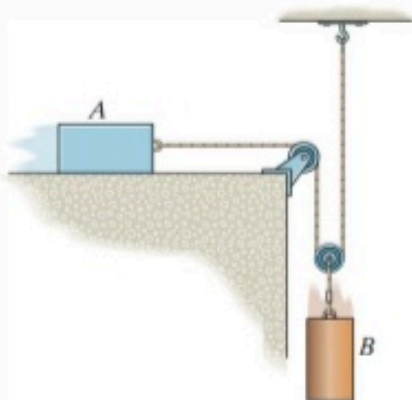
•13–21. Block B has a mass m and is released from rest when it is on top of cart A , which has a mass of $3m$. Determine the tension in cord CD needed to hold the cart from moving while B slides down A . Neglect friction.

13–22. Block B has a mass m and is released from rest when it is on top of cart A , which has a mass of $3m$. Determine the tension in cord CD needed to hold the cart from moving while B slides down A . The coefficient of kinetic friction between A and B is μ_k .



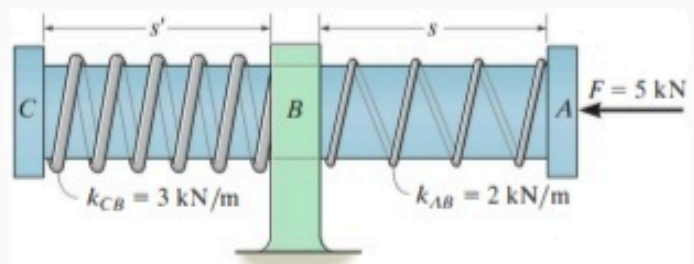
Probs. 13–21/22

***13–20.** The 10-lb block A travels to the right at $v_A = 2$ ft/s at the instant shown. If the coefficient of kinetic friction is $\mu_k = 0.2$ between the surface and A , determine the velocity of A when it has moved 4 ft. Block B has a weight of 20 lb.



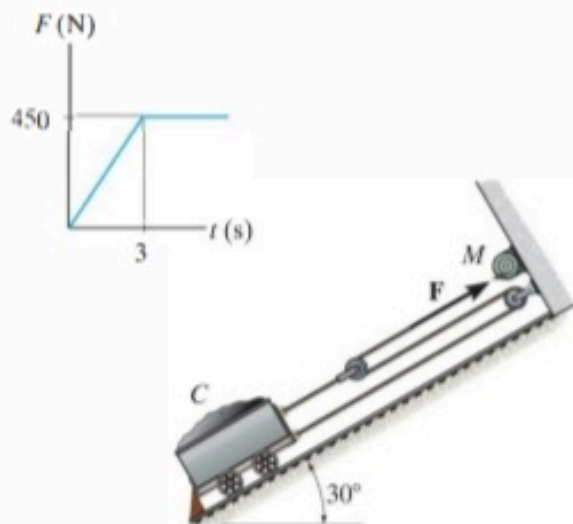
Prob. 13–20

13–23. The 2-kg shaft CA passes through a smooth journal bearing at B . Initially, the springs, which are coiled loosely around the shaft, are unstretched when no force is applied to the shaft. In this position $s = s' = 250$ mm and the shaft is at rest. If a horizontal force of $F = 5$ kN is applied, determine the speed of the shaft at the instant $s = 50$ mm, $s' = 450$ mm. The ends of the springs are attached to the bearing at B and the caps at C and A .



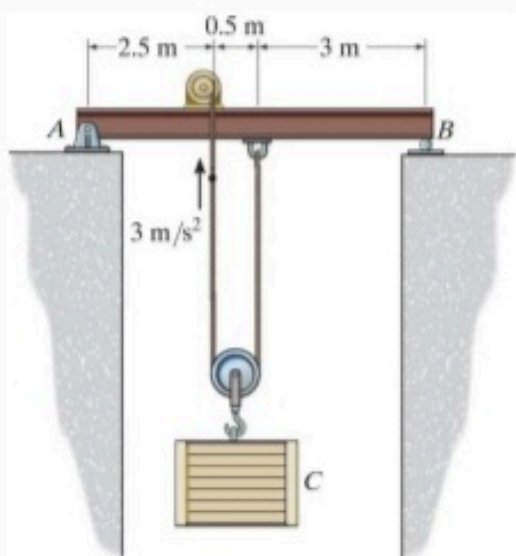
Prob. 13–23

***13–24.** If the force of the motor M on the cable is shown in the graph, determine the velocity of the cart when $t = 3$ s. The load and cart have a mass of 200 kg and the car starts from rest.



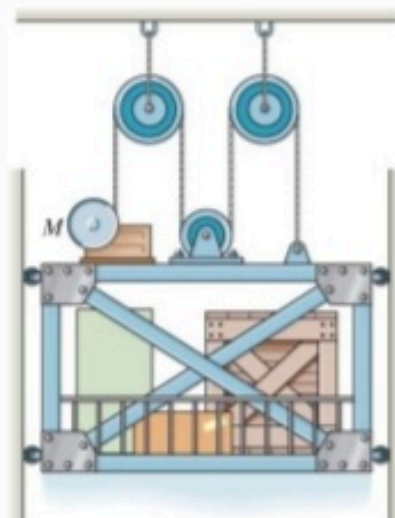
Prob. 13–24

•13–25. If the motor draws in the cable with an acceleration of 3 m/s^2 , determine the reactions at the supports A and B . The beam has a uniform mass of 30 kg/m , and the crate has a mass of 200 kg . Neglect the mass of the motor and pulleys.



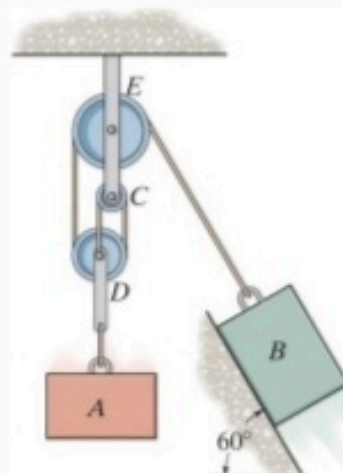
Prob. 13–25

13–26. A freight elevator, including its load, has a mass of 500 kg . It is prevented from rotating by the track and wheels mounted along its sides. When $t = 2$ s, the motor M draws in the cable with a speed of 6 m/s , measured relative to the elevator. If it starts from rest, determine the constant acceleration of the elevator and the tension in the cable. Neglect the mass of the pulleys, motor, and cables.



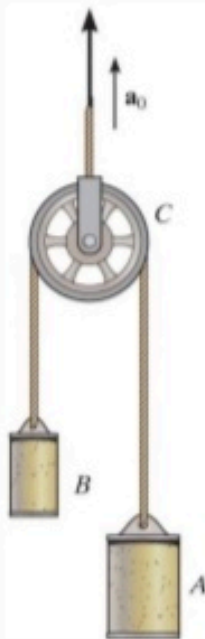
Prob. 13–26

13–27. Determine the required mass of block A so that when it is released from rest it moves the 5-kg block B a distance of 0.75 m up along the smooth inclined plane in $t = 2$ s. Neglect the mass of the pulleys and cords.



Prob. 13–27

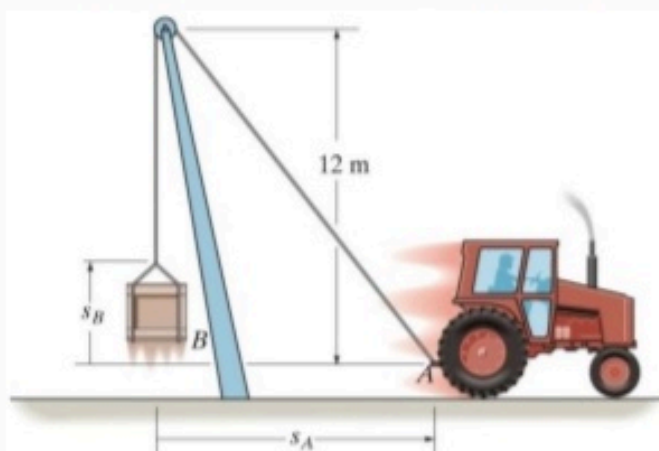
***13–28.** Blocks A and B have a mass of m_A and m_B , where $m_A > m_B$. If pulley C is given an acceleration of a_0 , determine the acceleration of the blocks. Neglect the mass of the pulley.



Prob. 13–28

***13–29.** The tractor is used to lift the 150-kg load B with the 24-m-long rope, boom, and pulley system. If the tractor travels to the right at a constant speed of 4 m/s, determine the tension in the rope when $s_A = 5$ m. When $s_A = 0$, $s_B = 0$.

13–30. The tractor is used to lift the 150-kg load B with the 24-m-long rope, boom, and pulley system. If the tractor travels to the right with an acceleration of 3 m/s² and has a velocity of 4 m/s at the instant $s_A = 5$ m, determine the tension in the rope at this instant. When $s_A = 0$, $s_B = 0$.



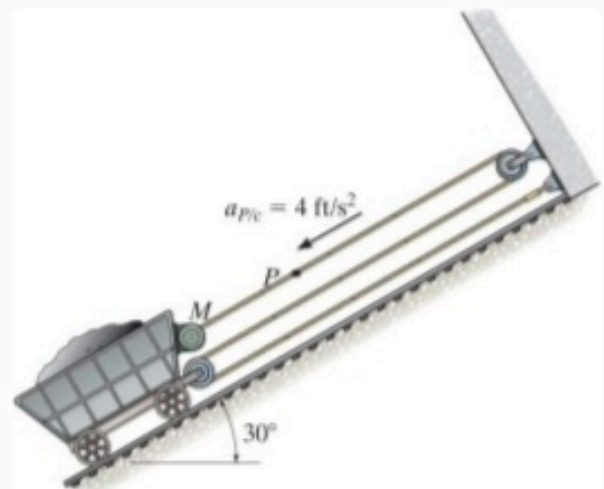
Probs. 13–29/30

13–31. The 75-kg man climbs up the rope with an acceleration of 0.25 m/s², measured relative to the rope. Determine the tension in the rope and the acceleration of the 80-kg block.



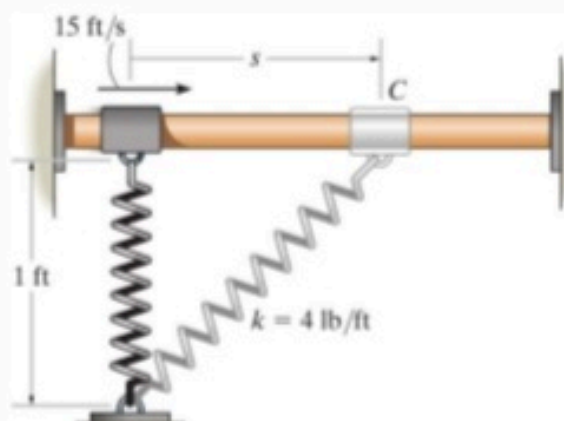
Prob. 13–31

***13–32.** Motor M draws in the cable with an acceleration of 4 ft/s², measured relative to the 200-lb mine car. Determine the acceleration of the car and the tension in the cable. Neglect the mass of the pulleys.



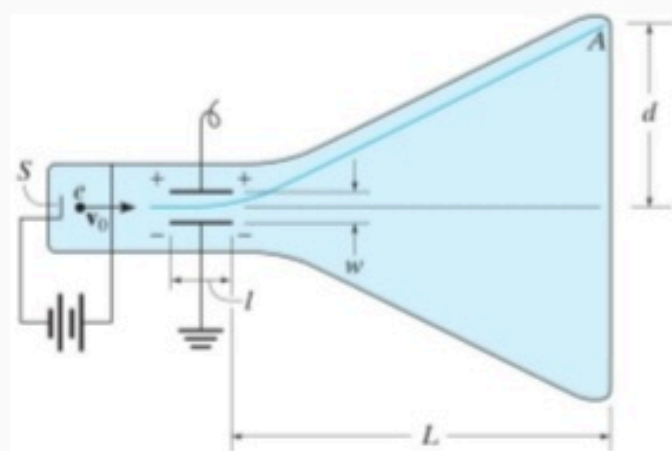
Prob. 13–32

- 13–33. The 2-lb collar C fits loosely on the smooth shaft. If the spring is unstretched when $s = 0$ and the collar is given a velocity of 15 ft/s, determine the velocity of the collar when $s = 1$ ft.



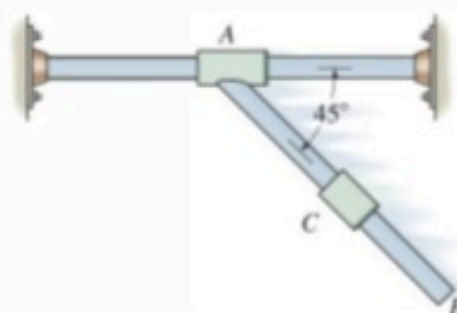
Prob. 13–33

- 13–34. In the cathode-ray tube, electrons having a mass m are emitted from a source point S and begin to travel horizontally with an initial velocity $\mathbf{v}_0$. While passing between the grid plates a distance l , they are subjected to a vertical force having a magnitude eV/w , where e is the charge of an electron, V the applied voltage acting across the plates, and w the distance between the plates. After passing clear of the plates, the electrons then travel in straight lines and strike the screen at A . Determine the deflection Δ of the electrons in terms of the dimensions of the voltage plate and tube. Neglect gravity which causes a slight vertical deflection when the electron travels from S to the screen, and the slight deflection between the plates.



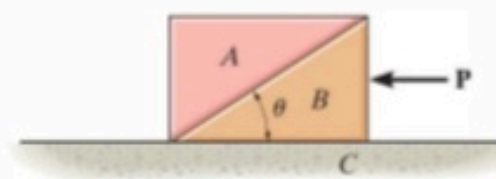
Prob. 13–34

- 13–35. The 2-kg collar C is free to slide along the smooth shaft AB . Determine the acceleration of collar C if (a) the shaft is fixed from moving, (b) collar A , which is fixed to the shaft AB , moves to the left at constant velocity along the horizontal guide, and (c) collar A is subjected to an acceleration of 2 m/s^2 to the left. In all cases, the motion occurs in the vertical plane.



Prob. 13–35

- *13–36. Blocks A and B each have a mass m . Determine the largest horizontal force $\mathbf{P}$ which can be applied to B so that A will not move relative to B . All surfaces are smooth.
- 13–37. Blocks A and B each have a mass m . Determine the largest horizontal force $\mathbf{P}$ which can be applied to B so that A will not slip on B . The coefficient of static friction between A and B is μ_s . Neglect any friction between B and C .



Probs. 13–36/37