

Kinetics of a Particle: Work and Energy

14

CHAPTER OBJECTIVES

- To develop the principle of work and energy and apply it to solve problems that involve force, velocity, and displacement.
- To study problems that involve power and efficiency.
- To introduce the concept of a conservative force and apply the theorem of conservation of energy to solve kinetic problems.

14.1 The Work of a Force

In this chapter, we will analyze motion of a particle using the concepts of work and energy. The resulting equation will be useful for solving problems that involve force, velocity, and displacement. Before we do this, however, we must first define the work of a force. Specifically, a force $\mathbf{F}$ will do *work* on a particle only when the particle undergoes a *displacement in the direction of the force*. For example, if the force $\mathbf{F}$ in Fig. 14–1 causes the particle to move along the path s from position $\mathbf{r}$ to a new position $\mathbf{r}'$, the displacement is then $d\mathbf{r} = \mathbf{r}' - \mathbf{r}$. The magnitude of $d\mathbf{r}$ is ds , the length of the differential segment along the path. If the angle between the tails of $d\mathbf{r}$ and $\mathbf{F}$ is θ , Fig. 14–1, then the work done by $\mathbf{F}$ is a *scalar quantity*, defined by

$$dU = F ds \cos \theta$$

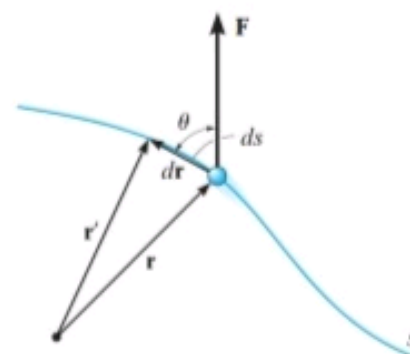


Fig. 14–1

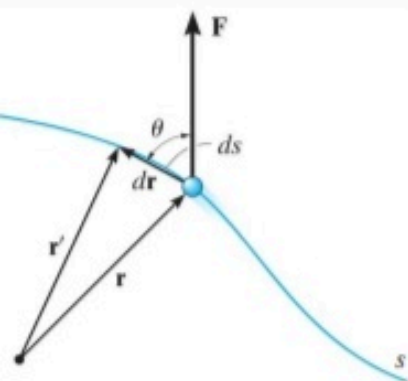


Fig. 14-1

By definition of the dot product (see Eq. B-14) this equation can also be written as

$$dU = \mathbf{F} \cdot d\mathbf{r}$$

This result may be interpreted in one of two ways: either as the product of F and the component of displacement $ds \cos \theta$ in the direction of the force, or as the product of ds and the component of force, $F \cos \theta$, in the direction of displacement. Note that if $0^\circ \leq \theta < 90^\circ$, then the force component and the displacement have the *same sense* so that the work is *positive*; whereas if $90^\circ < \theta \leq 180^\circ$, these vectors will have *opposite sense*, and therefore the work is *negative*. Also, $dU = 0$ if the force is *perpendicular* to displacement, since $\cos 90^\circ = 0$, or if the force is applied at a *fixed point*, in which case the displacement is zero.

The unit of work in SI units is the joule (J), which is the amount of work done by a one-newton force when it moves through a distance of one meter in the direction of the force ($1 \text{ J} = 1 \text{ N} \cdot \text{m}$). In the FPS system, work is measured in units of foot-pounds (ft · lb), which is the work done by a one-pound force acting through a distance of one foot in the direction of the force.*

Work of a Variable Force. If the particle acted upon by the force $\mathbf{F}$ undergoes a finite displacement along its path from $\mathbf{r}_1$ to $\mathbf{r}_2$ or s_1 to s_2 , Fig. 14-2a, the work of force $\mathbf{F}$ is determined by integration. Provided $\mathbf{F}$ and θ can be expressed as a function of position, then

$$U_{1-2} = \int_{\mathbf{r}_1}^{\mathbf{r}_2} \mathbf{F} \cdot d\mathbf{r} = \int_{s_1}^{s_2} F \cos \theta \, ds \quad (14-1)$$

Sometimes, this relation may be obtained by using experimental data to plot a graph of $F \cos \theta$ vs. s . Then the *area* under this graph bounded by s_1 and s_2 represents the total work, Fig. 14-2b.

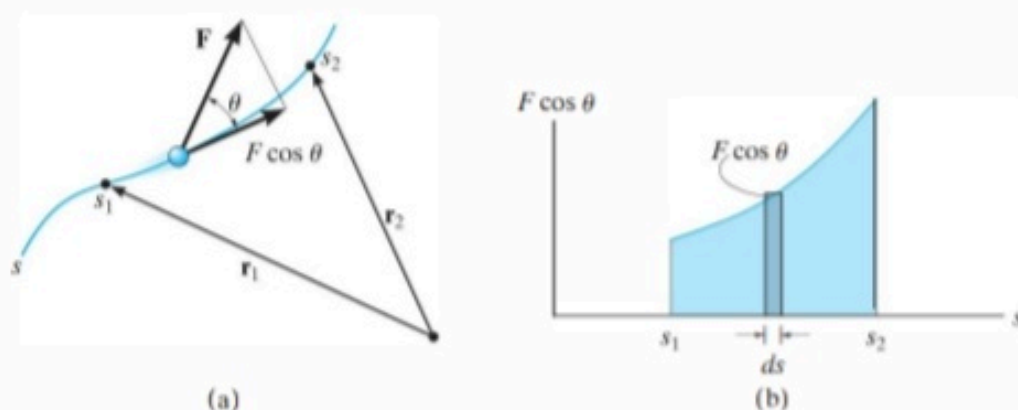


Fig. 14-2

*By convention, the units for the moment of a force or torque are written as lb · ft, to distinguish them from those used to signify work, ft · lb.



Fig. 14-3

Work of a Constant Force Moving Along a Straight Line.

If the force $\mathbf{F}_c$ has a constant magnitude and acts at a constant angle θ from its straight-line path, Fig. 14-3a, then the component of $\mathbf{F}_c$ in the direction of displacement is always $F_c \cos \theta$. The work done by $\mathbf{F}_c$ when the particle is displaced from s_1 to s_2 is determined from Eq. 14-1, in which case

$$U_{1-2} = F_c \cos \theta \int_{s_1}^{s_2} ds$$

or

$$U_{1-2} = F_c \cos \theta (s_2 - s_1) \quad (14-2)$$

Here the work of $\mathbf{F}_c$ represents the *area of the rectangle* in Fig. 14-3b.

Work of a Weight. Consider a particle of weight $\mathbf{W}$, which moves up along the path s shown in Fig. 14-4 from position s_1 to position s_2 . At an intermediate point, the displacement $d\mathbf{r} = dx\mathbf{i} + dy\mathbf{j} + dz\mathbf{k}$. Since $\mathbf{W} = -W\mathbf{j}$, applying Eq. 14-1 we have

$$\begin{aligned} U_{1-2} &= \int \mathbf{F} \cdot d\mathbf{r} = \int_{\mathbf{r}_1}^{\mathbf{r}_2} (-W\mathbf{j}) \cdot (dx\mathbf{i} + dy\mathbf{j} + dz\mathbf{k}) \\ &= \int_{y_1}^{y_2} -W dy = -W(y_2 - y_1) \end{aligned}$$

or

$$U_{1-2} = -W \Delta y \quad (14-3)$$

Thus, the work is independent of the path and is equal to the magnitude of the particle's weight times its vertical displacement. In the case shown in Fig. 14-4 the work is *negative*, since W is downward and Δy is upward. Note, however, that if the particle is displaced *downward* ($-\Delta y$), the work of the weight is *positive*. Why?

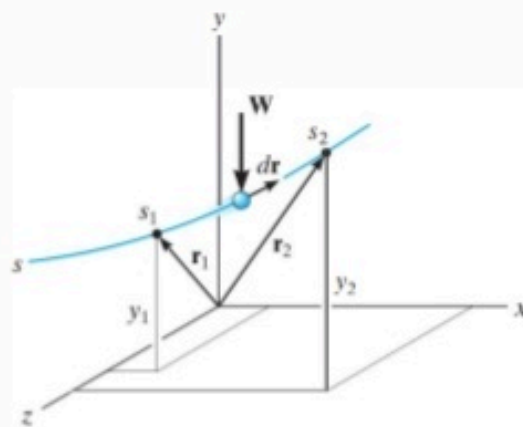


Fig. 14-4

Work of a Spring Force. If an elastic spring is elongated a distance ds , Fig. 14-5a, then the work done by the force that acts on the attached particle is $dU = -F_s ds = -ks ds$. The work is *negative* since $\mathbf{F}_s$ acts in the opposite sense to ds . If the particle displaces from s_1 to s_2 , the work of $\mathbf{F}_s$ is then

$$U_{1-2} = \int_{s_1}^{s_2} F_s ds = \int_{s_1}^{s_2} -ks ds$$

$$U_{1-2} = -\left(\frac{1}{2}ks_2^2 - \frac{1}{2}ks_1^2\right)$$

(14-4)

This work represents the trapezoidal area under the line $F_s = ks$, Fig. 14-5b.

A mistake in sign can be avoided when applying this equation if one simply notes the direction of the spring force acting on the particle and compares it with the sense of direction of displacement of the particle – if both are in the *same sense*, *positive work* results; if they are *opposite* to one another, the *work is negative*.

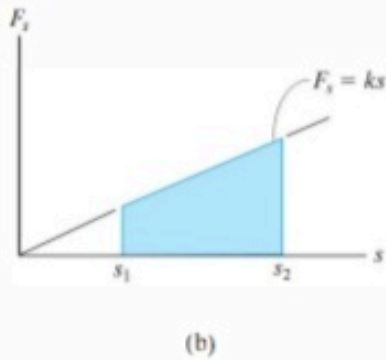
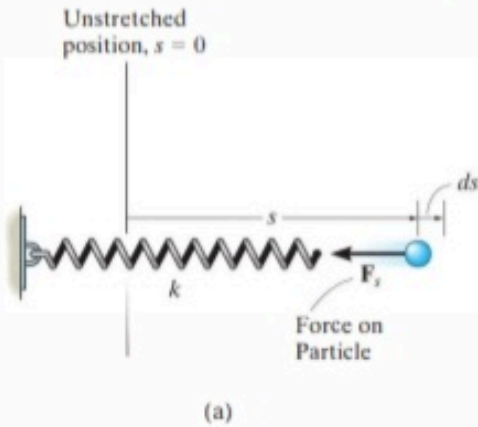
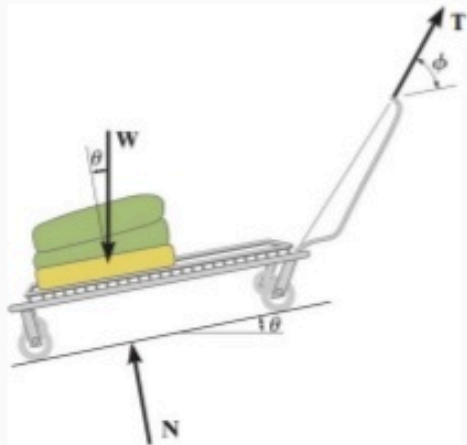


Fig. 14-5

The forces acting on the cart as it is pulled a distance s up the incline, are shown on its free-body diagram. The constant towing force $\mathbf{T}$ does positive work of $U_T = (T \cos \phi)s$, the weight does negative work of $U_W = -(W \sin \theta)s$, and the normal force $\mathbf{N}$ does no work since there is no displacement of this force along its line of action.



EXAMPLE 14.1

The 10-kg block shown in Fig. 14-6a rests on the smooth incline. If the spring is originally stretched 0.5 m, determine the total work done by all the forces acting on the block when a horizontal force $P = 400$ N pushes the block up the plane $s = 2$ m.

SOLUTION

First the free-body diagram of the block is drawn in order to account for all the forces that act on the block, Fig. 14-6b.

Horizontal Force P . Since this force is *constant*, the work is determined using Eq. 14-2. The result can be calculated as the force times the component of displacement in the direction of the force; i.e.,

$$U_P = 400 \text{ N} (2 \text{ m} \cos 30^\circ) = 692.8 \text{ J}$$

or the displacement times the component of force in the direction of displacement, i.e.,

$$U_P = 400 \text{ N} \cos 30^\circ (2 \text{ m}) = 692.8 \text{ J}$$

Spring Force F_s . In the initial position the spring is stretched $s_1 = 0.5$ m and in the final position it is stretched $s_2 = 0.5 \text{ m} + 2 \text{ m} = 2.5$ m. We require the work to be negative since the force and displacement are opposite to each other. The work of F_s is thus

$$U_s = -\left[\frac{1}{2}(30 \text{ N/m})(2.5 \text{ m})^2 - \frac{1}{2}(30 \text{ N/m})(0.5 \text{ m})^2\right] = -90 \text{ J}$$

Weight W . Since the weight acts in the opposite sense to its vertical displacement, the work is negative; i.e.,

$$U_W = -(98.1 \text{ N}) (2 \text{ m} \sin 30^\circ) = -98.1 \text{ J}$$

Note that it is also possible to consider the component of weight in the direction of displacement; i.e.,

$$U_W = -(98.1 \sin 30^\circ \text{ N}) (2 \text{ m}) = -98.1 \text{ J}$$

Normal Force N_B . This force does *no work* since it is *always* perpendicular to the displacement.

Total Work. The work of all the forces when the block is displaced 2 m is therefore

$$U_T = 692.8 \text{ J} - 90 \text{ J} - 98.1 \text{ J} = 505 \text{ J} \quad \text{Ans.}$$

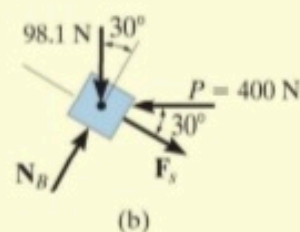
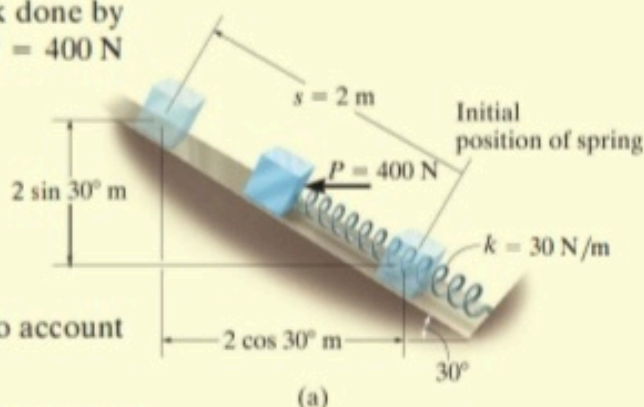
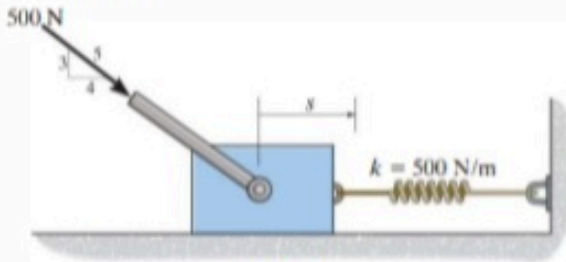


Fig. 14-6

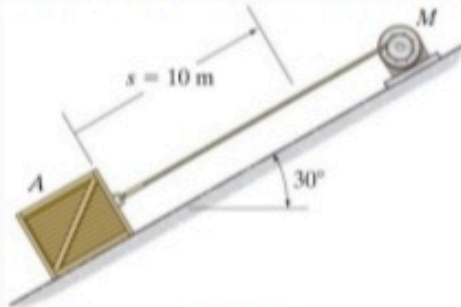
FUNDAMENTAL PROBLEMS

F14-1. The spring is placed between the wall and the 10-kg block. If the block is subjected to a force of $F = 500$ N, determine its velocity when $s = 0.5$ m. When $s = 0$, the block is at rest and the spring is uncompressed. The contact surface is smooth.



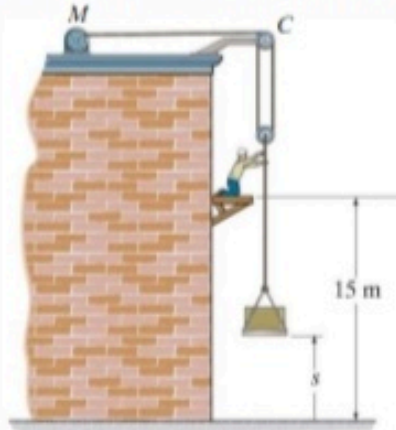
F14-1

F14-2. If the motor exerts a constant force of 300 N on the cable, determine the speed of the 20-kg crate when it travels $s = 10$ m up the plane, starting from rest. The coefficient of kinetic friction between the crate and the plane is $\mu_k = 0.3$.



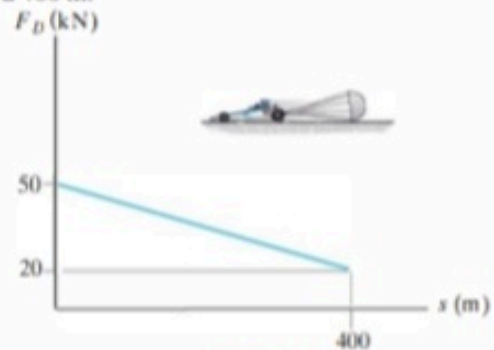
F14-2

F14-3. If the motor exerts a force of $F = (600 + 2s^2)$ N on the cable, determine the speed of the 100-kg crate when it rises to $s = 15$ m. The crate is initially at rest on the ground.



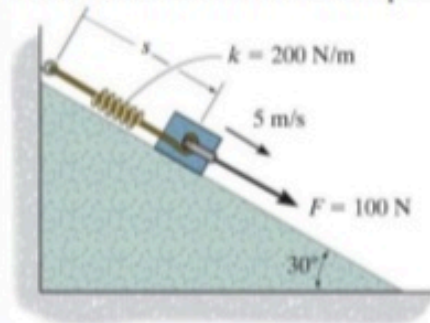
F14-3

F14-4. The 1.8-Mg dragster is traveling at 125 m/s when the engine is shut off and the parachute is released. If the drag force of the parachute can be approximated by the graph, determine the speed of the dragster when it has traveled 400 m.



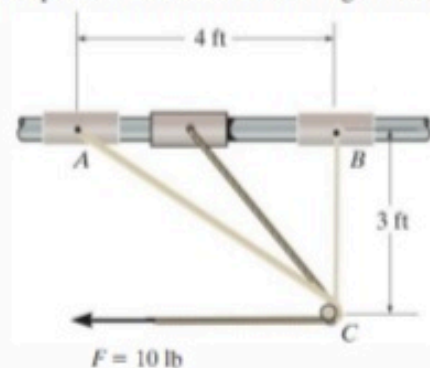
F14-4

F14-5. When $s = 0.6$ m, the spring is unstretched and the 10-kg block has a speed of 5 m/s down the smooth plane. Determine the distance s when the block stops.



F14-5

F14-6. The 5-lb collar is pulled by a cord that passes around a small peg at C. If the cord is subjected to a constant force of $F = 10$ lb, and the collar is at rest when it is at A, determine its speed when it reaches B. Neglect friction.



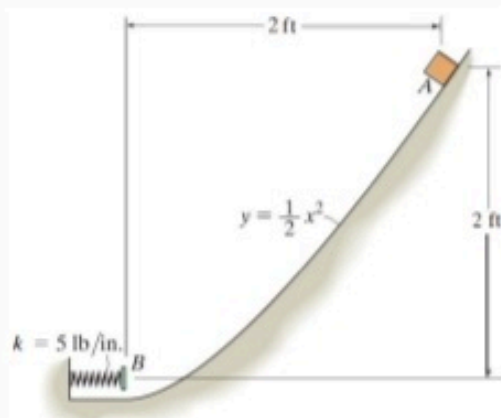
F14-6

14-6. When the driver applies the brakes of a light truck traveling 10 km/h, it skids 3 m before stopping. How far will the truck skid if it is traveling 80 km/h when the brakes are applied?



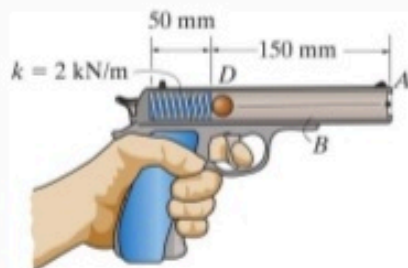
Prob. 14-6

14-7. The 6-lb block is released from rest at *A* and slides down the smooth parabolic surface. Determine the maximum compression of the spring.



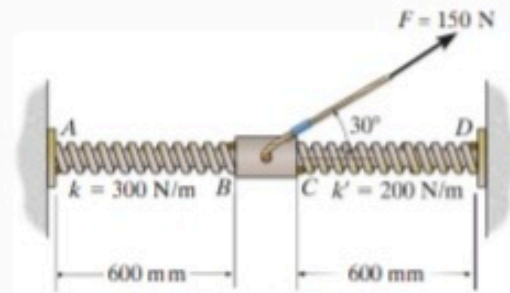
Prob. 14-7

***14-8.** The spring in the toy gun has an unstretched length of 100 mm. It is compressed and locked in the position shown. When the trigger is pulled, the spring unstretches 12.5 mm, and the 20-g ball moves along the barrel. Determine the speed of the ball when it leaves the gun. Neglect friction.



Prob. 14-8

***14-9.** Springs *AB* and *CD* have a stiffness of $k = 300$ N/m and $k' = 200$ N/m, respectively, and both springs have an unstretched length of 600 mm. If the 2-kg smooth collar starts from rest when the springs are unstretched, determine the speed of the collar when it has moved 200 mm.



Prob. 14-9

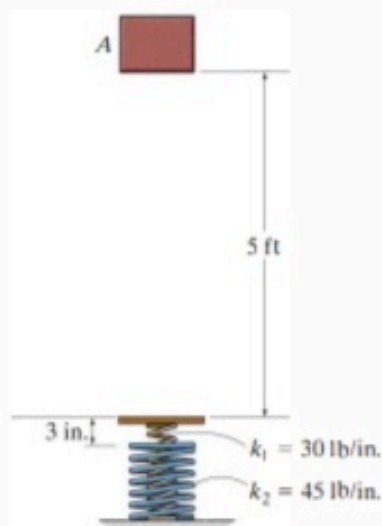
14-10. The 2-Mg car has a velocity of $v_1 = 100$ km/h when the driver sees an obstacle in front of the car. If it takes 0.75 s for him to react and lock the brakes, causing the car to skid, determine the distance the car travels before it stops. The coefficient of kinetic friction between the tires and the road is $\mu_k = 0.25$.

14-11. The 2-Mg car has a velocity of $v_1 = 100$ km/h when the driver sees an obstacle in front of the car. It takes 0.75 s for him to react and lock the brakes, causing the car to skid. If the car stops when it has traveled a distance of 175 m, determine the coefficient of kinetic friction between the tires and the road.



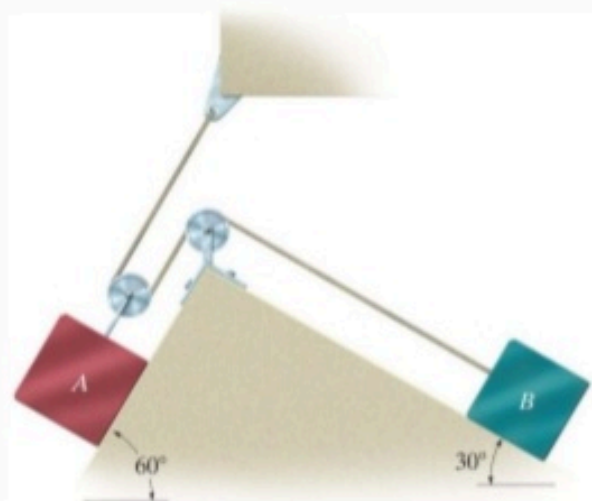
Probs. 14-10/11

***14-12.** The 10-lb block is released from rest at *A*. Determine the compression of each of the springs after the block strikes the platform and is brought momentarily to rest. Initially both springs are unstretched. Assume the platform has a negligible mass.



Prob. 14-12

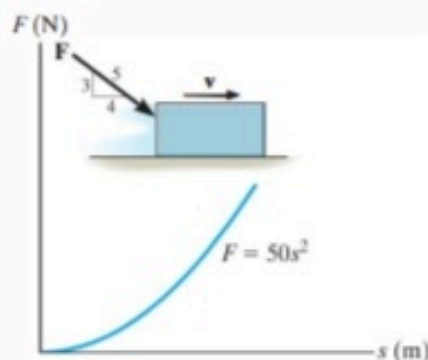
14-13. Determine the velocity of the 60-lb block *A* if the two blocks are released from rest and the 40-lb block *B* moves 2 ft up the incline. The coefficient of kinetic friction between both blocks and the inclined planes is $\mu_k = 0.10$.



Prob. 14-13

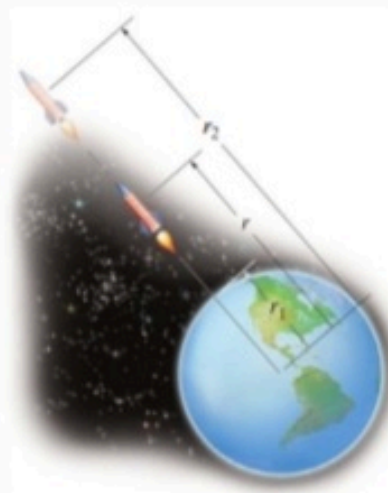
14-14. The force **F**, acting in a constant direction on the 20-kg block, has a magnitude which varies with the position *s* of the block. Determine how far the block slides before its velocity becomes 5 m/s. When *s* = 0 the block is moving to the right at 2 m/s. The coefficient of kinetic friction between the block and surface is $\mu_k = 0.3$.

14-15. The force **F**, acting in a constant direction on the 20-kg block, has a magnitude which varies with position *s* of the block. Determine the speed of the block after it slides 3 m. When *s* = 0 the block is moving to the right at 2 m/s. The coefficient of kinetic friction between the block and surface is $\mu_k = 0.3$.



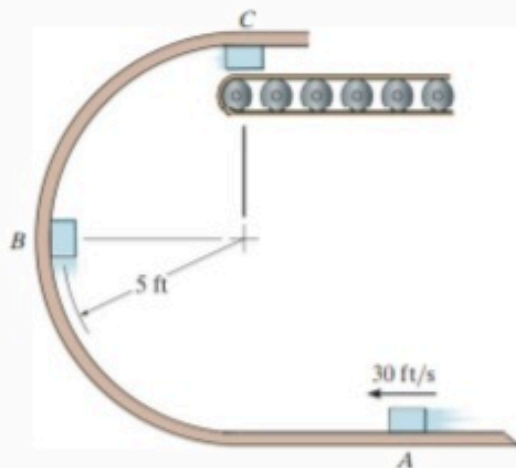
Prob. 14-15

14-16. A rocket of mass *m* is fired vertically from the surface of the earth, i.e., at $r = r_1$. Assuming no mass is lost as it travels upward, determine the work it must do against gravity to reach a distance r_2 . The force of gravity is $F = GM_em/r^2$ (Eq. 13-1), where M_e is the mass of the earth and *r* the distance between the rocket and the center of the earth.



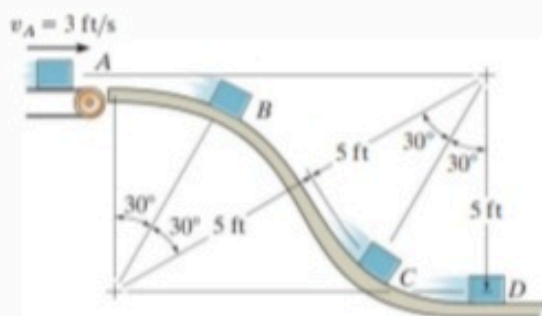
Prob. 14-16

4-22. The 2-lb box slides on the smooth circular ramp. If the box has a velocity of 30 ft/s at A , determine the velocity of the box and normal force acting on the ramp when the box is located at B and C . Assume the radius of curvature of the path at C is still 5 ft.



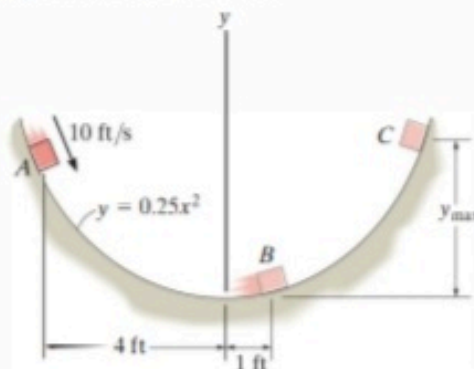
Prob. 14-22

4-23. Packages having a weight of 50 lb are delivered to the chute at $v_A = 3$ ft/s using a conveyor belt. Determine their speeds when they reach points B , C , and D . Also calculate the normal force of the chute on the packages at B and C . Neglect friction and the size of the packages.



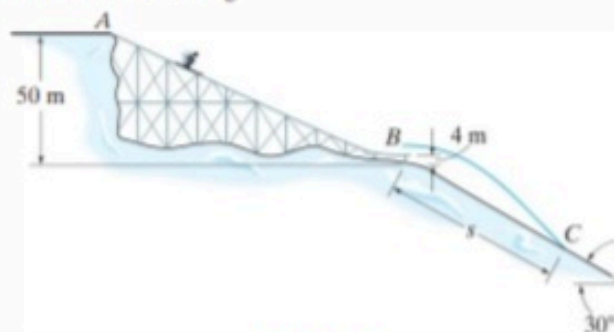
Prob. 14-23

***14-24.** The 2-lb block slides down the smooth parabolic surface, such that when it is at A it has a speed of 10 ft/s. Determine the magnitude of the block's velocity and acceleration when it reaches point B , and the maximum height $y_{\max}$ reached by the block.



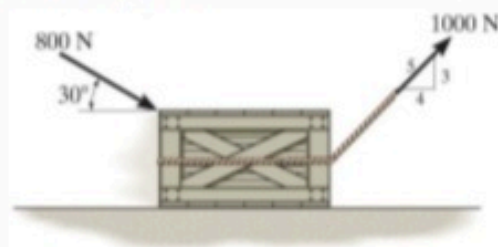
Prob. 14-24

***14-25.** The skier starts from rest at A and travels down the ramp. If friction and air resistance can be neglected, determine his speed v_B when he reaches B . Also, find the distance s to where he strikes the ground at C , if he makes the jump traveling horizontally at B . Neglect the skier's size. He has a mass of 70 kg.



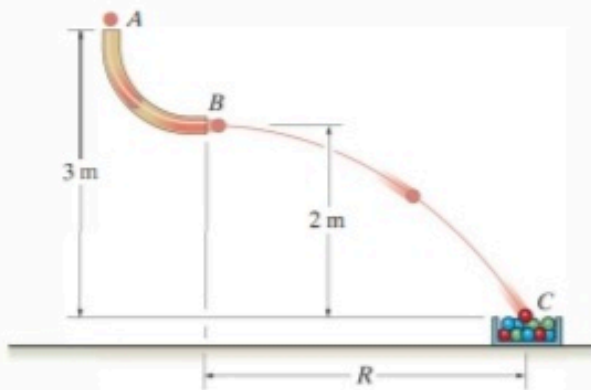
Prob. 14-25

14-26. The crate, which has a mass of 100 kg, is subjected to the action of the two forces. If it is originally at rest, determine the distance it slides in order to attain a speed of 6 m/s. The coefficient of kinetic friction between the crate and the surface is $\mu_k = 0.2$.



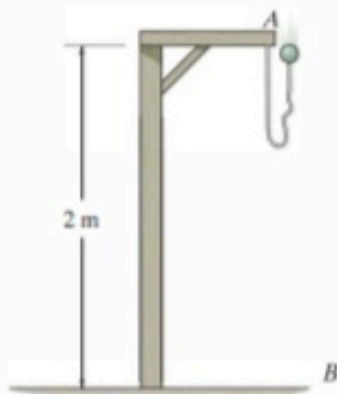
Prob. 14-26

14-31. Marbles having a mass of 5 g fall from rest at A through the glass tube and accumulate in the can at C . Determine the placement R of the can from the end of the tube and the speed at which the marbles fall into the can. Neglect the size of the can.



Prob. 14-31

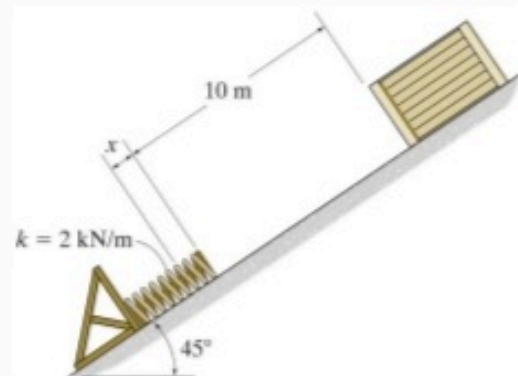
***14-32.** The ball has a mass of 0.5 kg and is suspended from a rubber band having an unstretched length of 1 m and a stiffness $k = 50 \text{ N/m}$. If the support at A to which the rubber band is attached is 2 m from the floor, determine the greatest speed the ball can have at A so that it does not touch the floor when it reaches its lowest point B . Neglect the size of the ball and the mass of the rubber band.



Prob. 14-32

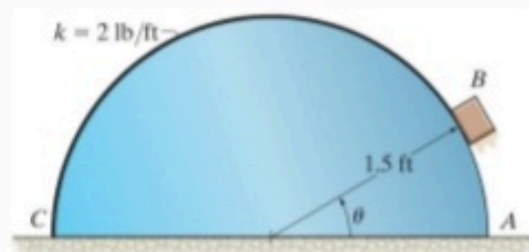
•14-33. If the coefficient of kinetic friction between the 100-kg crate and the plane is $\mu_k = 0.25$, determine the compression x of the spring required to bring the crate momentarily to rest. Initially the spring is unstretched and the crate is at rest.

14-34. If the coefficient of kinetic friction between the 100-kg crate and the plane is $\mu_k = 0.25$, determine the speed of the crate at the instant the compression of the spring is $x = 1.5 \text{ m}$. Initially the spring is unstretched and the crate is at rest.



Probs. 14-33/34

14-35. A 2-lb block rests on the smooth semicylindrical surface. An elastic cord having a stiffness $k = 2 \text{ lb/ft}$ is attached to the block at B and to the base of the semicylinder at point C . If the block is released from rest at A ($\theta = 0^\circ$), determine the unstretched length of the cord so that the block begins to leave the semicylinder at the instant $\theta = 45^\circ$. Neglect the size of the block.



Prob. 14-35