

12.6 Motion of a Projectile

The free-flight motion of a projectile is often studied in terms of its rectangular components. To illustrate the kinematic analysis, consider a projectile launched at point (x_0, y_0) , with an initial velocity of $\mathbf{v}_0$, having components $(\mathbf{v}_0)_x$ and $(\mathbf{v}_0)_y$, Fig. 12–20. When air resistance is neglected, the only force acting on the projectile is its weight, which causes the projectile to have a *constant downward acceleration* of approximately $a_c = g = 9.81 \text{ m/s}^2$ or $g = 32.2 \text{ ft/s}^2$.*

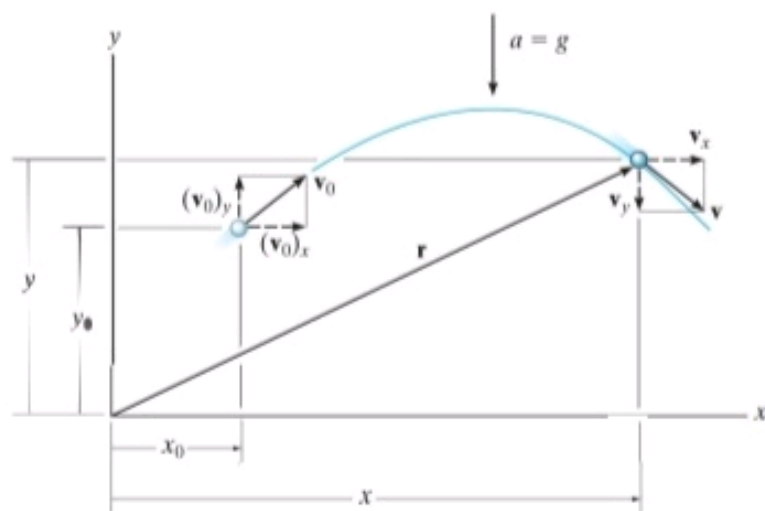


Fig. 12–20

Horizontal Motion. Since $a_x = 0$, application of the constant acceleration equations, 12–4 to 12–6, yields

$$\begin{aligned} (\rightarrow) \quad v &= v_0 + a_c t; & v_x &= (v_0)_x \\ (\rightarrow) \quad x &= x_0 + v_0 t + \frac{1}{2} a_c t^2; & x &= x_0 + (v_0)_x t \\ (\rightarrow) \quad v^2 &= v_0^2 + 2a_c(x - x_0); & v_x &= (v_0)_x \end{aligned}$$

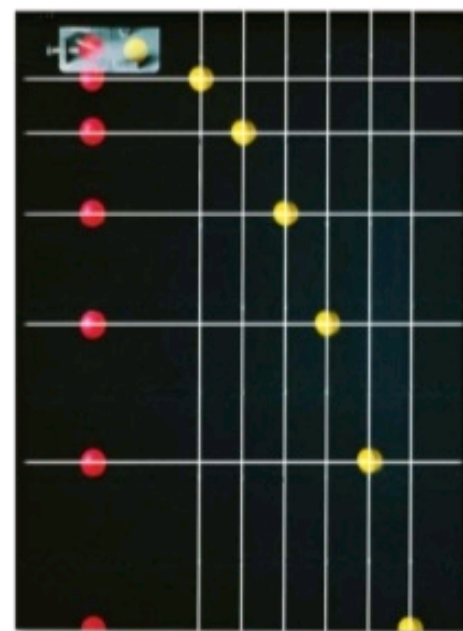
The first and last equations indicate that *the horizontal component of velocity always remains constant during the motion*.

Vertical Motion. Since the positive y axis is directed upward, then $a_y = -g$. Applying Eqs. 12–4 to 12–6, we get

$$\begin{aligned} (+\uparrow) \quad v &= v_0 + a_c t; & v_y &= (v_0)_y - gt \\ (+\uparrow) \quad y &= y_0 + v_0 t + \frac{1}{2} a_c t^2; & y &= y_0 + (v_0)_y t - \frac{1}{2} gt^2 \\ (+\uparrow) \quad v^2 &= v_0^2 + 2a_c(y - y_0); & v_y^2 &= (v_0)_y^2 - 2g(y - y_0) \end{aligned}$$

Recall that the last equation can be formulated on the basis of eliminating the time t from the first two equations, and therefore *only two of the above three equations are independent of one another*.

* This assumes that the earth's gravitational field does not vary with altitude.



Each picture in this sequence is taken after the same time interval. The red ball falls from rest, whereas the yellow ball is given a horizontal velocity when released. Both balls accelerate downward at the same rate, and so they remain at the same elevation at any instant. This acceleration causes the difference in elevation between the balls to increase between successive photos. Also, note the horizontal distance between successive photos of the yellow ball is constant since the velocity in the horizontal direction remains constant.

To summarize, problems involving the motion of a projectile can have at most three unknowns since only three independent equations can be written; that is, *one* equation in the *horizontal direction* and *two* in the *vertical direction*. Once v_x and v_y are obtained, the resultant velocity v , which is *always tangent* to the path, can be determined by the *vector sum* as shown in Fig. 12–20.

Procedure for Analysis

Coordinate System.

- Establish the fixed x, y coordinate axes and sketch the trajectory of the particle. Between any *two points* on the path specify the given problem data and identify the *three unknowns*. In all cases the acceleration of gravity acts downward and equals 9.81 m/s^2 or 32.2 ft/s^2 . The particle's initial and final velocities should be represented in terms of their x and y components.
- Remember that positive and negative position, velocity, and acceleration components always act in accordance with their associated coordinate directions.

Kinematic Equations.

- Depending upon the known data and what is to be determined, a choice should be made as to which three of the following four equations should be applied between the two points on the path to obtain the most direct solution to the problem.

Horizontal Motion.

- The *velocity* in the horizontal or x direction is *constant*, i.e., $v_x = (v_0)_x$, and

$$x = x_0 + (v_0)_x t$$

Vertical Motion.

- In the vertical or y direction *only two* of the following three equations can be used for solution.

$$v_y = (v_0)_y + a_c t$$

$$y = y_0 + (v_0)_y t + \frac{1}{2} a_c t^2$$

$$v_y^2 = (v_0)_y^2 + 2a_c(y - y_0)$$

For example, if the particle's final velocity v_y is not needed, then the first and third of these equations will not be useful.



Gravel falling off the end of this conveyor belt follows a path that can be predicted using the equations of constant acceleration. In this way the location of the accumulated pile can be determined. Rectangular coordinates are used for the analysis since the acceleration is only in the vertical direction.

EXAMPLE 12.11

A sack slides off the ramp, shown in Fig. 12–21, with a horizontal velocity of 12 m/s. If the height of the ramp is 6 m from the floor, determine the time needed for the sack to strike the floor and the range R where sacks begin to pile up.

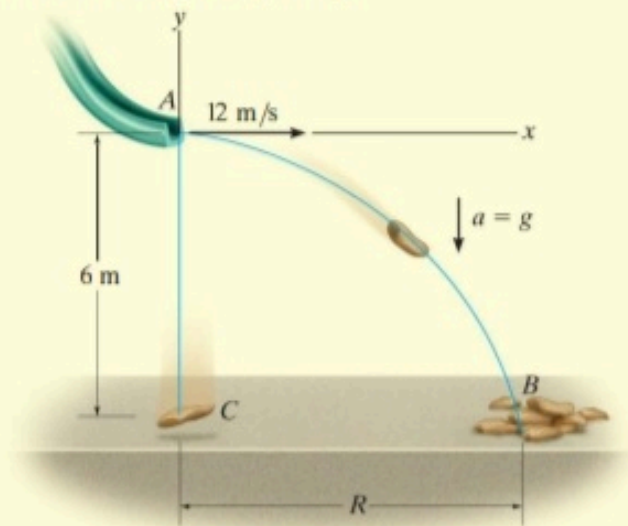


Fig. 12–21

SOLUTION

Coordinate System. The origin of coordinates is established at the beginning of the path, point A, Fig. 12–21. The initial velocity of a sack has components $(v_A)_x = 12 \text{ m/s}$ and $(v_A)_y = 0$. Also, between points A and B the acceleration is $a_y = -9.81 \text{ m/s}^2$. Since $(v_B)_x = (v_A)_x = 12 \text{ m/s}$, the three unknowns are $(v_B)_y$, R , and the time of flight t_{AB} . Here we do not need to determine $(v_B)_y$.

Vertical Motion. The vertical distance from A to B is known, and therefore we can obtain a direct solution for t_{AB} by using the equation

$$\begin{aligned} (+\uparrow) \quad y_B &= y_A + (v_A)_y t_{AB} + \frac{1}{2} a_y t_{AB}^2 \\ -6 \text{ m} &= 0 + 0 + \frac{1}{2} (-9.81 \text{ m/s}^2) t_{AB}^2 \\ t_{AB} &= 1.11 \text{ s} \end{aligned} \quad \text{Ans.}$$

Horizontal Motion. Since t_{AB} has been calculated, R is determined as follows:

$$\begin{aligned} (\rightarrow) \quad x_B &= x_A + (v_A)_x t_{AB} \\ R &= 0 + 12 \text{ m/s} (1.11 \text{ s}) \\ R &= 13.3 \text{ m} \end{aligned} \quad \text{Ans.}$$

NOTE: The calculation for t_{AB} also indicates that if a sack were released *from rest* at A, it would take the same amount of time to strike the floor at C, Fig. 12–21.

EXAMPLE 12.12

The chipping machine is designed to eject wood chips at $v_O = 25$ ft/s as shown in Fig. 12–22. If the tube is oriented at 30° from the horizontal, determine how high, h , the chips strike the pile if at this instant they land on the pile 20 ft from the tube.

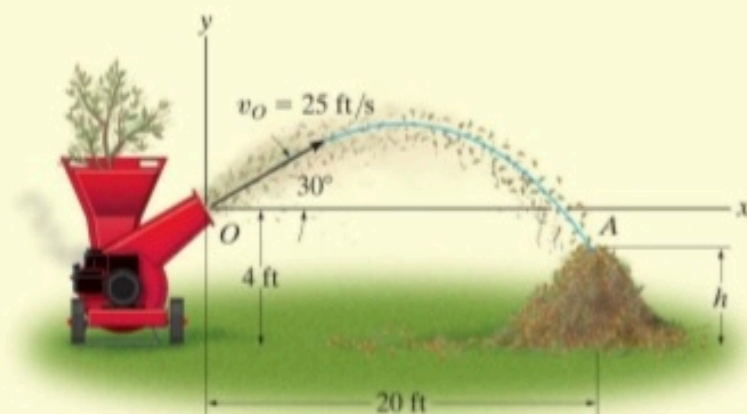


Fig. 12–22

SOLUTION

Coordinate System. When the motion is analyzed between points O and A , the three unknowns are the height h , time of flight t_{OA} , and vertical component of velocity $(v_A)_y$. [Note that $(v_A)_x = (v_O)_x$.] With the origin of coordinates at O , Fig. 12–22, the initial velocity of a chip has components of

$$(v_O)_x = (25 \cos 30^\circ) \text{ ft/s} = 21.65 \text{ ft/s} \rightarrow$$

$$(v_O)_y = (25 \sin 30^\circ) \text{ ft/s} = 12.5 \text{ ft/s} \uparrow$$

Also, $(v_A)_x = (v_O)_x = 21.65 \text{ ft/s}$ and $a_y = -32.2 \text{ ft/s}^2$. Since we do not need to determine $(v_A)_y$, we have

Horizontal Motion.

$$\begin{aligned} (\rightarrow) \quad x_A &= x_O + (v_O)_x t_{OA} \\ 20 \text{ ft} &= 0 + (21.65 \text{ ft/s}) t_{OA} \\ t_{OA} &= 0.9238 \text{ s} \end{aligned}$$

Vertical Motion. Relating t_{OA} to the initial and final elevations of a chip, we have

$$\begin{aligned} (+\uparrow) \quad y_A &= y_O + (v_O)_y t_{OA} + \frac{1}{2} a_y t_{OA}^2 \\ (h - 4 \text{ ft}) &= 0 + (12.5 \text{ ft/s})(0.9238 \text{ s}) + \frac{1}{2}(-32.2 \text{ ft/s}^2)(0.9238 \text{ s})^2 \\ h &= 1.81 \text{ ft} \end{aligned} \quad \text{Ans.}$$

NOTE: We can determine $(v_A)_y$ by using $(v_A)_y = (v_O)_y + a_y t_{OA}$.

EXAMPLE 12.13

The track for this racing event was designed so that riders jump off the slope at 30° , from a height of 1 m. During a race it was observed that the rider shown in Fig. 12–23a remained in mid air for 1.5 s. Determine the speed at which he was traveling off the ramp, the horizontal distance he travels before striking the ground, and the maximum height he attains. Neglect the size of the bike and rider.



(a)

SOLUTION

Coordinate System. As shown in Fig. 12–23b, the origin of the coordinates is established at A . Between the end points of the path AB the three unknowns are the initial speed v_A , range R , and the vertical component of velocity $(v_B)_y$.

Vertical Motion. Since the time of flight and the vertical distance between the ends of the path are known, we can determine v_A .

$$\begin{aligned}
 (+\uparrow) \quad y_B &= y_A + (v_A)_y t_{AB} + \frac{1}{2} a_c t_{AB}^2 \\
 -1 \text{ m} &= 0 + v_A \sin 30^\circ (1.5 \text{ s}) + \frac{1}{2} (-9.81 \text{ m/s}^2) (1.5 \text{ s})^2 \\
 v_A &= 13.38 \text{ m/s} = 13.4 \text{ m/s} \quad \text{Ans.}
 \end{aligned}$$

Horizontal Motion. The range R can now be determined.

$$\begin{aligned}
 (\rightarrow) \quad x_B &= x_A + (v_A)_x t_{AB} \\
 R &= 0 + 13.38 \cos 30^\circ \text{ m/s} (1.5 \text{ s}) \\
 &= 17.4 \text{ m} \quad \text{Ans.}
 \end{aligned}$$

In order to find the maximum height h we will consider the path AC , Fig. 12–23b. Here the three unknowns are the time of flight t_{AC} , the horizontal distance from A to C , and the height h . At the maximum height $(v_C)_y = 0$, and since v_A is known, we can determine h directly without considering t_{AC} using the following equation.

$$\begin{aligned}
 (v_C)_y^2 &= (v_A)_y^2 + 2a_c[y_C - y_A] \\
 0^2 &= (13.38 \sin 30^\circ \text{ m/s})^2 + 2(-9.81 \text{ m/s}^2)[(h - 1 \text{ m}) - 0] \\
 h &= 3.28 \text{ m} \quad \text{Ans.}
 \end{aligned}$$

NOTE: Show that the bike will strike the ground at B with a velocity having components of

$$(v_B)_x = 11.6 \text{ m/s} \rightarrow, \quad (v_B)_y = 8.02 \text{ m/s} \downarrow$$

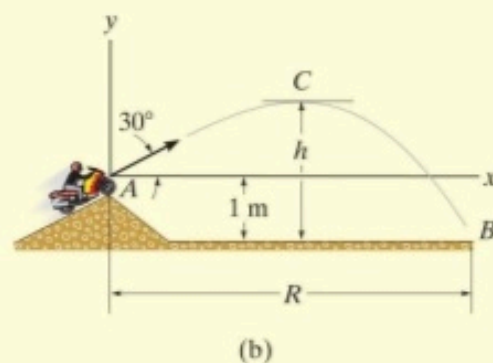
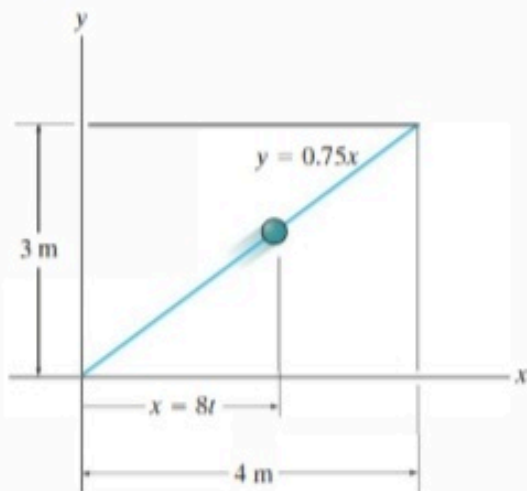


Fig. 12–23

FUNDAMENTAL PROBLEMS

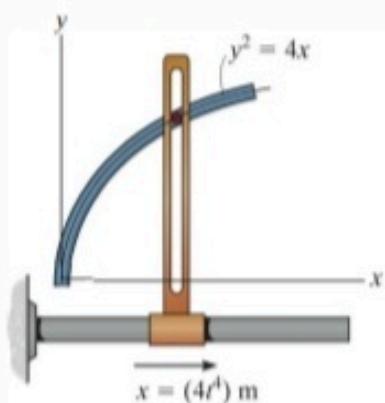
F12-15. If the x and y components of a particle's velocity are $v_x = (32t)$ m/s and $v_y = 8$ m/s, determine the equation of the path $y = f(x)$. $x = 0$ and $y = 0$ when $t = 0$.

F12-16. A particle is traveling along the straight path. If its position along the x axis is $x = (8t)$ m, where t is in seconds, determine its speed when $t = 2$ s.



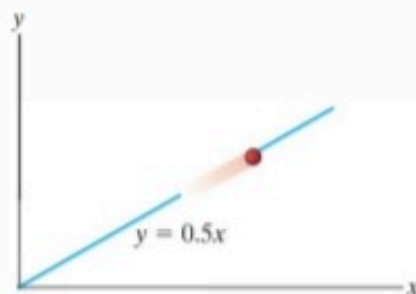
F12-16

F12-17. A particle is constrained to travel along the path. If $x = (4t^4)$ m, where t is in seconds, determine the magnitude of the particle's velocity and acceleration when $t = 0.5$ s.



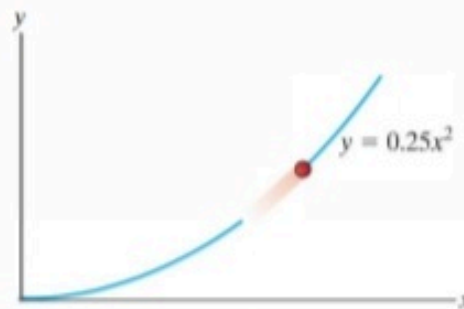
F12-17

F12-18. A particle travels along a straight-line path $y = 0.5x$. If the x component of the particle's velocity is $v_x = (2t^2)$ m/s, where t is in seconds, determine the magnitude of the particle's velocity and acceleration when $t = 4$ s.



F12-18

F12-19. A particle is traveling along the parabolic path $y = 0.25x^2$. If $x = (2t^2)$ m, where t is in seconds, determine the magnitude of the particle's velocity and acceleration when $t = 2$ s.



F12-19

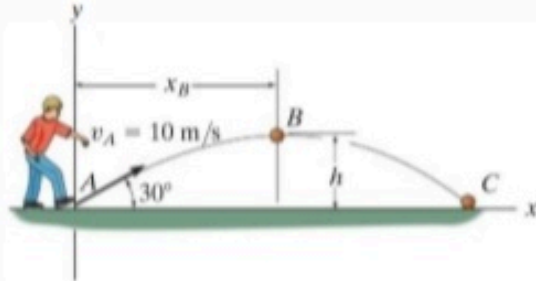
F12-20. The position of a box sliding down the spiral can be described by $\mathbf{r} = [2 \sin(2t)\mathbf{i} + 2 \cos t\mathbf{j} - 2t^2\mathbf{k}]$ ft, where t is in seconds and the arguments for the sine and cosine are in radians. Determine the velocity and acceleration of the box when $t = 2$ s.



F12-20

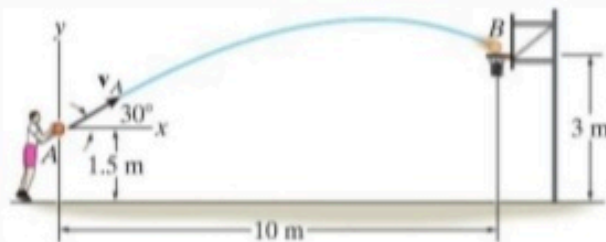
F12-21. The ball is kicked from point A with the initial velocity $v_A = 10 \text{ m/s}$. Determine the maximum height h it reaches.

F12-22. The ball is kicked from point A with the initial velocity $v_A = 10 \text{ m/s}$. Determine the range R , and the speed when the ball strikes the ground.



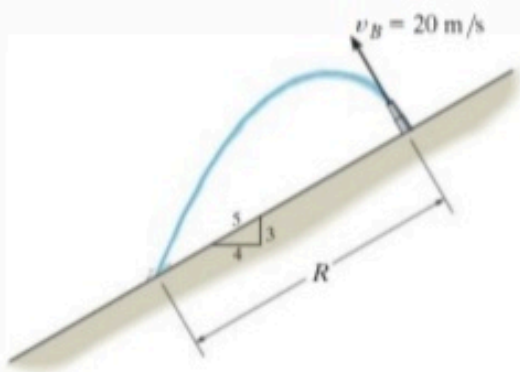
F12-21/22

F12-23. Determine the speed at which the basketball at A must be thrown at the angle of 30° so that it makes it to the basket at B .



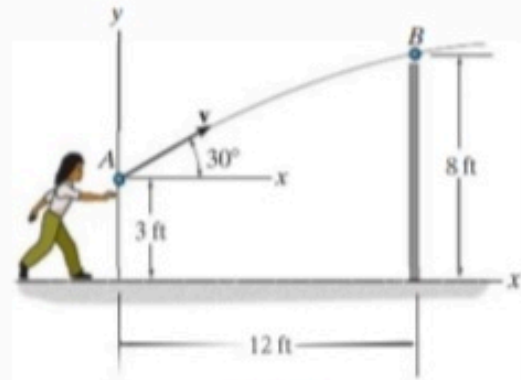
F12-23

F12-24. Water is sprayed at an angle of 90° from the slope at 20 m/s . Determine the range R .



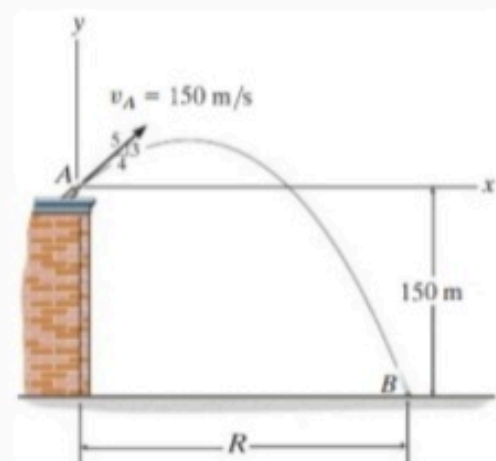
F12-24

F12-25. A ball is thrown from A . If it is required to clear the wall at B , determine the minimum magnitude of its initial velocity v_A .



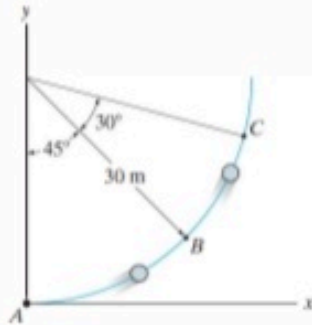
F12-25

F12-26. A projectile is fired with an initial velocity of $v_A = 150 \text{ m/s}$ off the roof of the building. Determine the range R where it strikes the ground at B .



F12-26

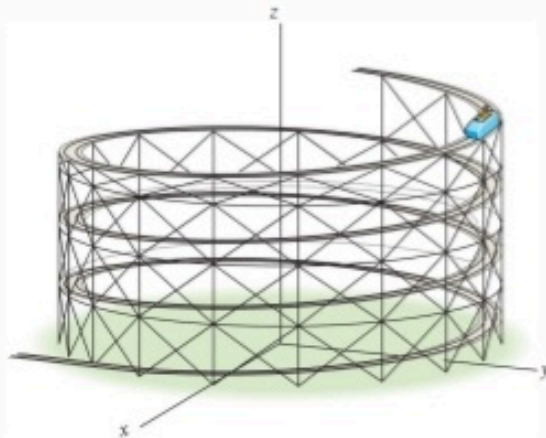
•12–81. A particle travels along the circular path from A to B in 1 s. If it takes 3 s for it to go from A to C , determine its *average velocity* when it goes from B to C .



Prob. 12–81

12–82. A car travels east 2 km for 5 minutes, then north 3 km for 8 minutes, and then west 4 km for 10 minutes. Determine the total distance traveled and the magnitude of displacement of the car. Also, what is the magnitude of the average velocity and the average speed?

12–83. The roller coaster car travels down the helical path at constant speed such that the parametric equations that define its position are $x = c \sin kt$, $y = c \cos kt$, $z = h - bt$, where c , h , and b are constants. Determine the magnitudes of its velocity and acceleration.

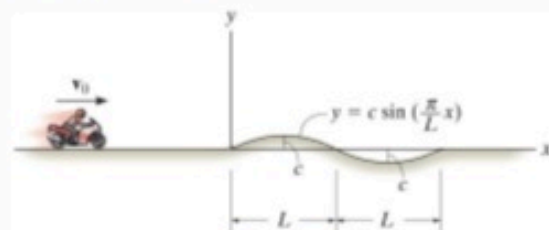


Prob. 12–83

*12–84. The path of a particle is defined by $y^2 = 4kx$, and the component of velocity along the y axis is $v_y = ct$, where both k and c are constants. Determine the x and y components of acceleration when $y = y_0$.

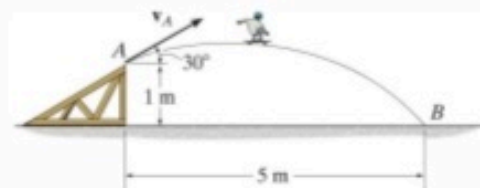
*12–85. A particle moves along the curve $y = x - (x^2/400)$, where x and y are in ft. If the velocity component in the x direction is $v_x = 2$ ft/s and remains constant, determine the magnitudes of the velocity and acceleration when $x = 20$ ft.

12–86. The motorcycle travels with constant speed v_0 along the path that, for a short distance, takes the form of a sine curve. Determine the x and y components of its velocity at any instant on the curve.



Prob. 12–86

12–87. The skateboard rider leaves the ramp at A with an initial velocity v_A at a 30° angle. If he strikes the ground at B , determine v_A and the time of flight.



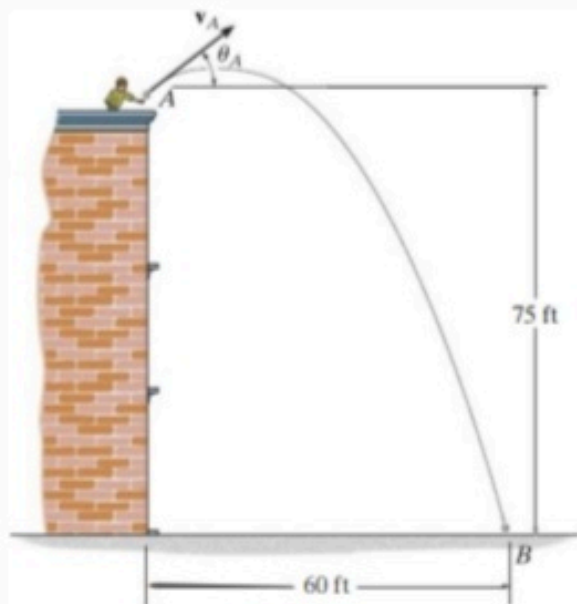
Prob. 12–87

*12–88. The pitcher throws the baseball horizontally with a speed of 140 ft/s from a height of 5 ft. If the batter is 60 ft away, determine the time for the ball to arrive at the batter and the height h at which it passes the batter.



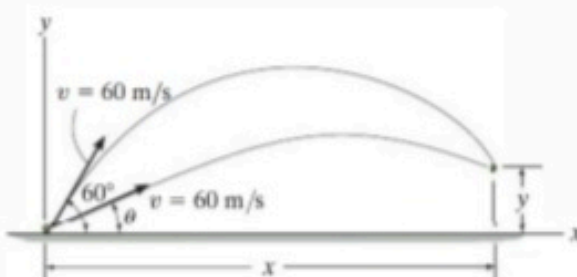
Prob. 12–88

12 •12–89. The ball is thrown off the top of the building. If it strikes the ground at B in 3 s, determine the initial velocity v_A and the inclination angle θ_A at which it was thrown. Also, find the magnitude of the ball's velocity when it strikes the ground.



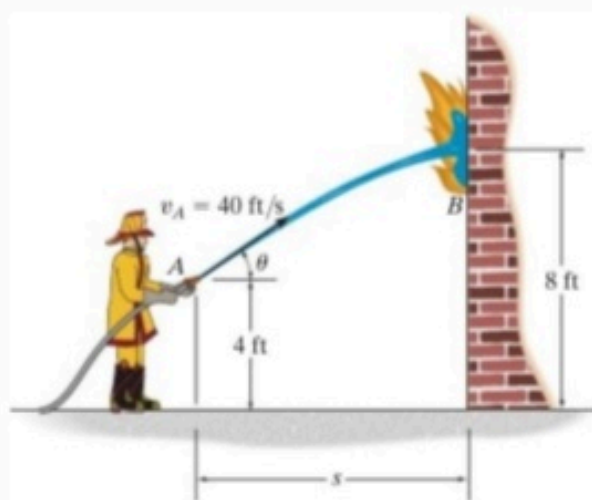
Prob. 12–89

12–90. A projectile is fired with a speed of $v = 60$ m/s at an angle of 60° . A second projectile is then fired with the same speed 0.5 s later. Determine the angle θ of the second projectile so that the two projectiles collide. At what position (x, y) will this happen?



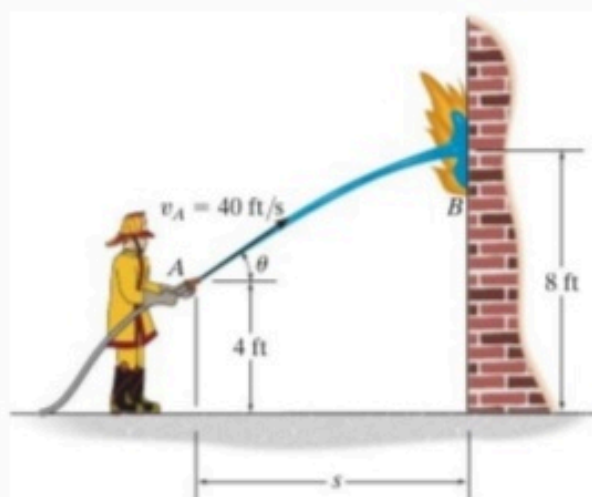
Prob. 12–90

12–91. The fireman holds the hose at an angle $\theta = 30^\circ$ with horizontal, and the water is discharged from the hose at A with a speed of $v_A = 40$ ft/s. If the water stream strikes the building at B , determine his two possible distances s from the building.



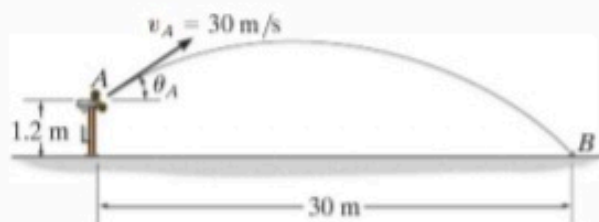
Prob. 12–91

*12–92. Water is discharged from the hose with a speed of 40 ft/s. Determine the two possible angles θ the fireman can hold the hose so that the water strikes the building at B . Take $s = 20$ ft.



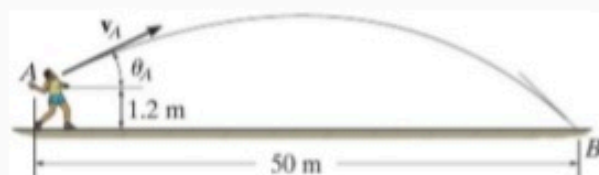
Prob. 12–92

•12-93. The pitching machine is adjusted so that the baseball is launched with a speed of $v_A = 30 \text{ m/s}$. If the ball strikes the ground at B , determine the two possible angles θ_A at which it was launched.



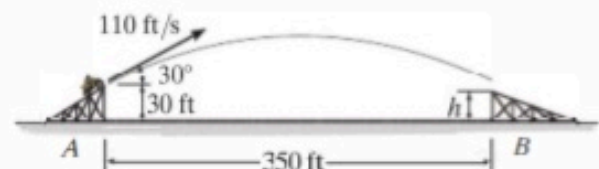
Prob. 12-93

12-94. It is observed that the time for the ball to strike the ground at B is 2.5 s. Determine the speed v_A and angle θ_A at which the ball was thrown.



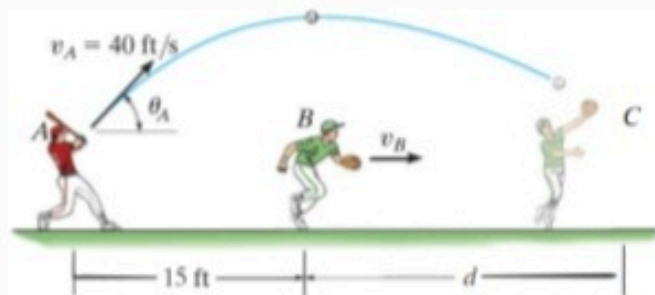
Prob. 12-94

12-95. If the motorcycle leaves the ramp traveling at 110 ft/s, determine the height h ramp B must have so that the motorcycle lands safely.



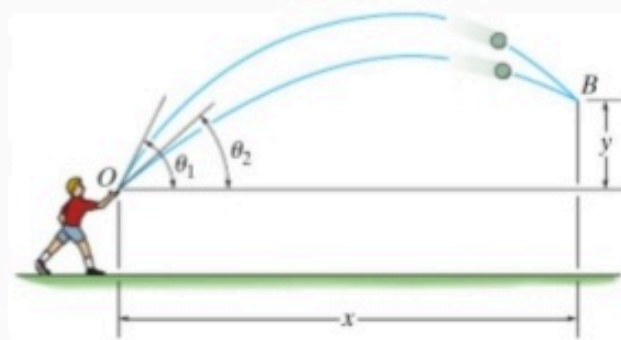
Prob. 12-95

*12-96. The baseball player A hits the baseball with $v_A = 40 \text{ ft/s}$ and $\theta_A = 60^\circ$. When the ball is directly above of player B he begins to run under it. Determine the constant speed v_B and the distance d at which B must run in order to make the catch at the same elevation at which the ball was hit.



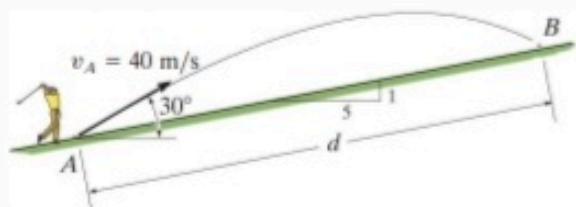
Prob. 12-96

•12-97. A boy throws a ball at O in the air with a speed v_0 at an angle θ_1 . If he then throws another ball with the same speed v_0 at an angle $\theta_2 < \theta_1$, determine the time between the throws so that the balls collide in mid air at B .



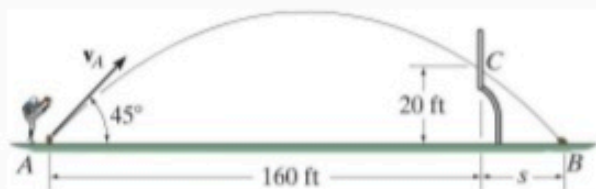
Prob. 12-97

12-98. The golf ball is hit at A with a speed of $v_A = 40 \text{ m/s}$ and directed at an angle of 30° with the horizontal as shown. Determine the distance d where the ball strikes the slope at B .



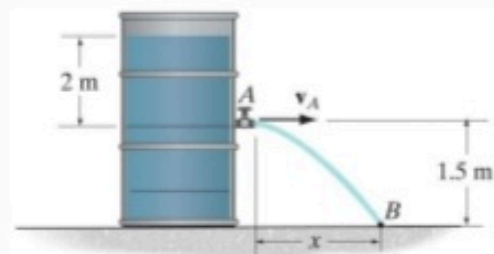
Prob. 12-98

12-99. If the football is kicked at the 45° angle, determine its minimum initial speed v_A so that it passes over the goal post at C . At what distance s from the goal post will the football strike the ground at B ?



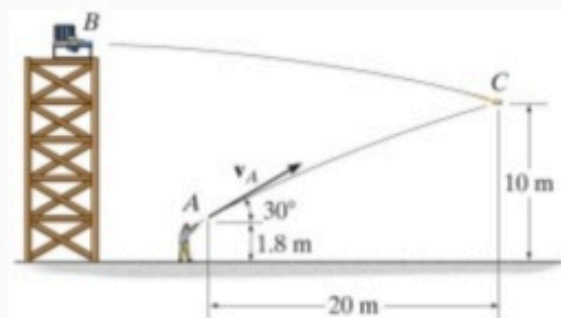
Prob. 12-99

***12-100.** The velocity of the water jet discharging from the orifice can be obtained from $v = \sqrt{2gh}$, where $h = 2 \text{ m}$ is the depth of the orifice from the free water surface. Determine the time for a particle of water leaving the orifice to reach point B and the horizontal distance x where it hits the surface.



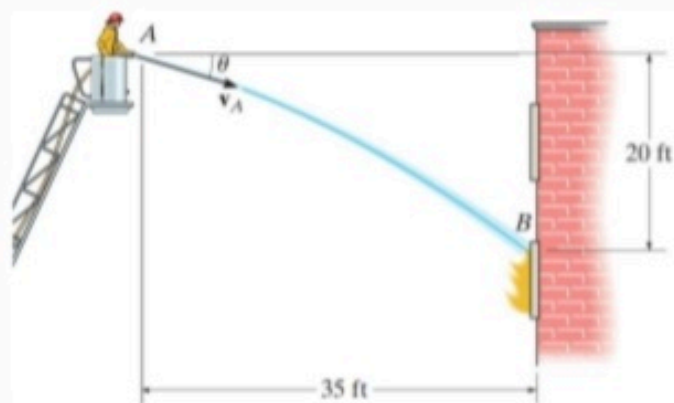
Prob. 12-100

***12-101.** A projectile is fired from the platform at B . The shooter fires his gun from point A at an angle of 30° . Determine the muzzle speed of the bullet if it hits the projectile at C .



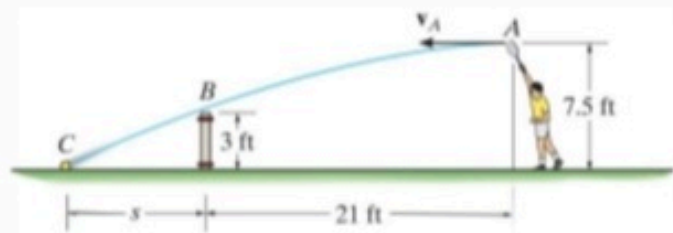
Prob. 12-101

12-107. The fireman wishes to direct the flow of water from his hose to the fire at B . Determine two possible angles θ_1 and θ_2 at which this can be done. Water flows from the hose at $v_A = 80 \text{ ft/s}$.



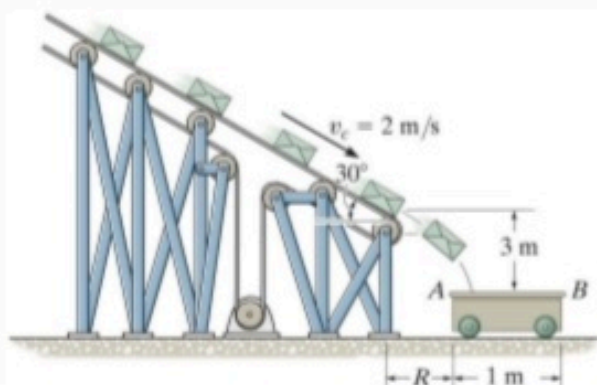
Prob. 12-107

12-109. Determine the horizontal velocity v_A of a tennis ball at A so that it just clears the net at B . Also, find the distance s where the ball strikes the ground.



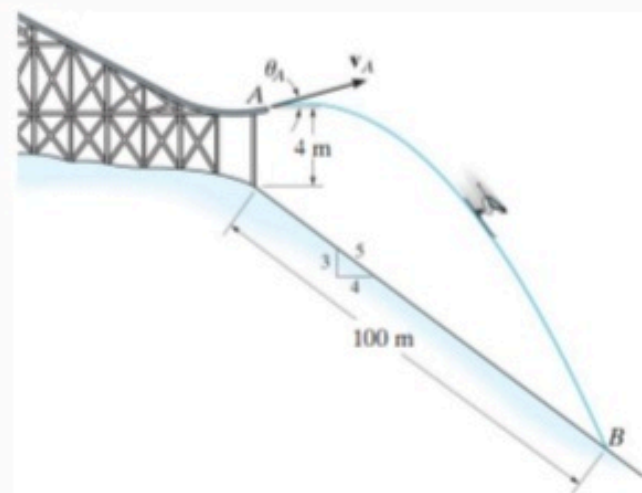
Prob. 12-109

12-108. Small packages traveling on the conveyor belt fall off into a 1-m-long loading car. If the conveyor is running at a constant speed of $v_c = 2 \text{ m/s}$, determine the smallest and largest distance R at which the end A of the car may be placed from the conveyor so that the packages enter the car.



Prob. 12-108

12-110. It is observed that the skier leaves the ramp A at an angle $\theta_A = 25^\circ$ with the horizontal. If he strikes the ground at B , determine his initial speed v_A and the time of flight t_{AB} .



Prob. 12-110