

First Class

Circuit Theorems

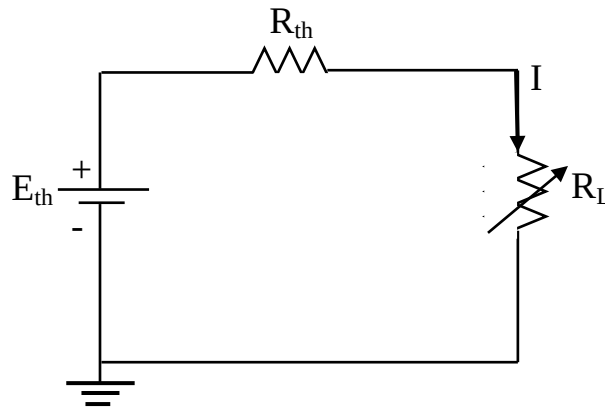
4) Maximum Power Transfer Theorem

Introduction

The Maximum power transfer theorem states that: A load will receive maximum power from a linear bilateral dc network when its total resistive value is exactly equal to the Thevenin resistance of the network as “seen” by the load.

For the network shown, the maximum power will be delivered to the load when

$$R_{th} = R_L$$



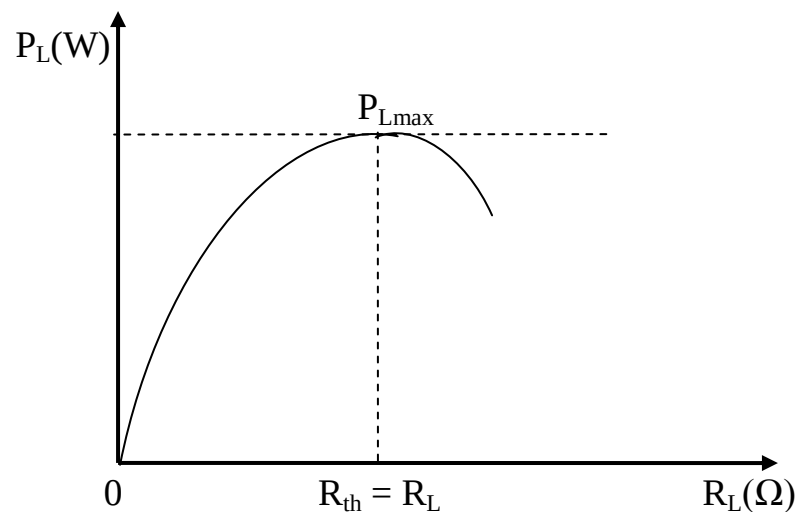
$$I = \frac{E_{th}}{R_{th} + R_L}$$

$$P_L = I^2 R_L = \left(\frac{E_{th}}{R_{th} + R_L} \right)^2 R_L$$

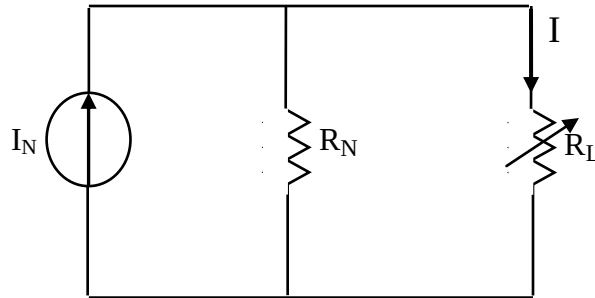
$$\therefore P_{Lmax} \Rightarrow \text{at } R_{th} = R_L$$

$$P_{Lmax} = \left(\frac{E_{th}}{2R_{th}} \right)^2 R_{th} = \frac{E_{th}^2}{4R_{th}}$$

$$\therefore P_{Lmax} = \frac{E_{th}^2}{4R_{th}}$$



When dealing with Norton equivalent circuit, maximum power transfer takes place when: $R_N = R_L$



$$R_N = R_L$$

$$P_L = I^2 R_L$$

$$\text{Where } I = I_N \times \frac{R_N}{R_N + R_L} \quad \text{using current divider rule}$$

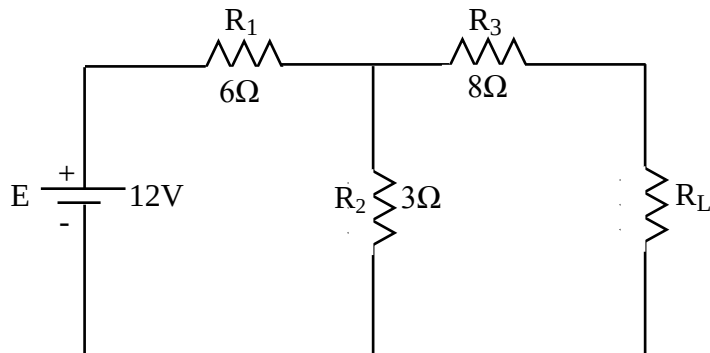
$$P_L = \left(I_N \times \frac{R_N}{R_N + R_L} \right)^2 \cdot R_L$$

$$P_{L\max} \Rightarrow \text{at } R_N = R_L$$

$$\therefore P_{L\max} = \left(I_N \times \frac{R_N}{2R_N} \right)^2 \cdot R_N$$

$$\therefore P_{L\max} = \frac{I_N^2 R_N}{4}$$

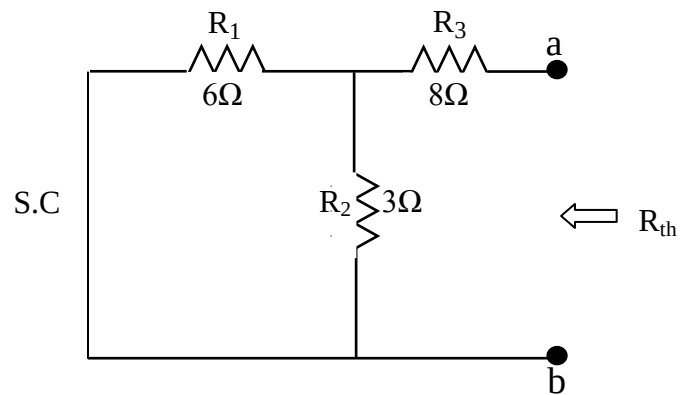
Example1: For the circuit shown, determine the value of R_L for maximum power transfer, and calculate the power delivered under these conditions.



Solution:

- R_{th} ?

$$\therefore R_{th} = (R_1 // R_2) + R_3 = \frac{(6)(3)}{6 + 3} + 8 = 10\Omega$$



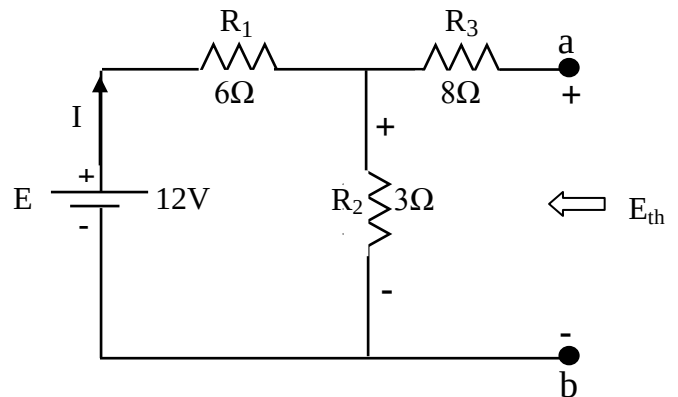
For Maximum power transfer, the value of $R_L = R_{th} = 10\Omega$

- E_{th} ?

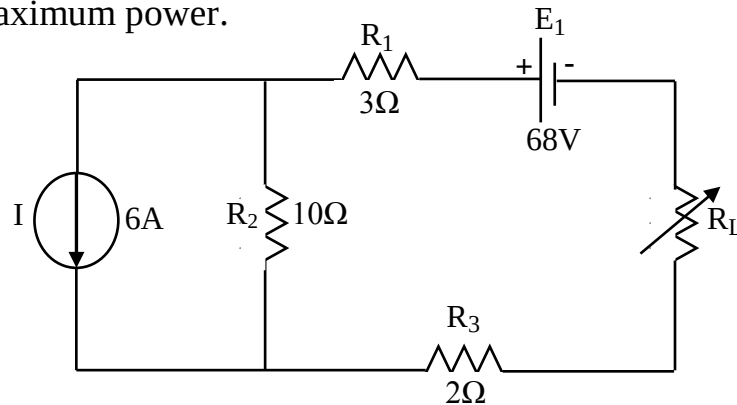
$$E_{th} = E \frac{R_2}{R_1 + R_2}$$

$$= 12 \frac{3}{6 + 3} = 4V$$

$$\therefore P_{Lmax} = \frac{E_{th}^2}{4R_{th}} = \frac{(4)^2}{4(10)} = 0.4W$$



Example2: Find the value of R_L in the circuit shown, for maximum power transfer to R_L and determine the maximum power.



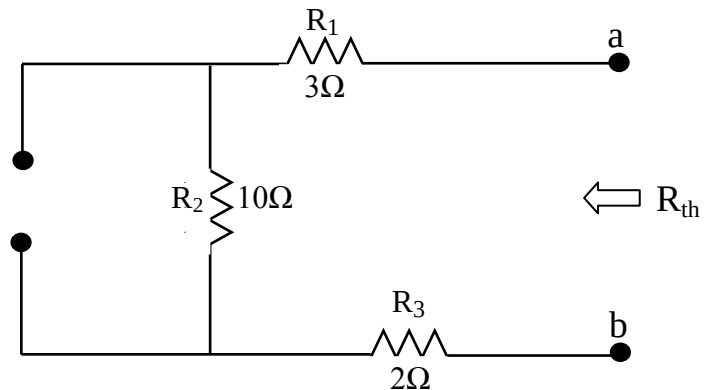
Solution:

- R_{th} ?

$$R_{th} = 3 + 10 + 2 = 15\Omega$$

For maximum power transfer

$$R_L = R_{th} = 15\Omega$$

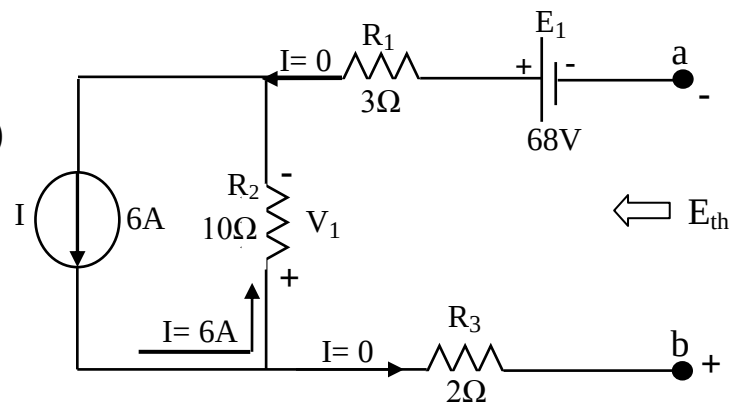


- E_{th} ?

$$E_{th} = E_1 + V_1$$

$$E_{th} = E_1 + IR_2 = 68 + 6(10)$$

$$E_{th} = 128V$$



$$\therefore P_{Lmax} = \frac{E_{th}^2}{4R_{th}} = \frac{(128)^2}{4(15)} = 273.07W$$

OR

The current source can be converted into a voltage source:

$$R_{th} = R_1 + R_2 + R_3$$

$$R_{th} = 3 + 10 + 2 = 15\Omega$$

For maximum power transfer

$$R_L = R_{th} = 15\Omega$$

$$\therefore E_{th} = E_1 + E_2$$

$$E_{th} = 68 + 60 = 128V$$

