
First Class

Nodal Voltage Method

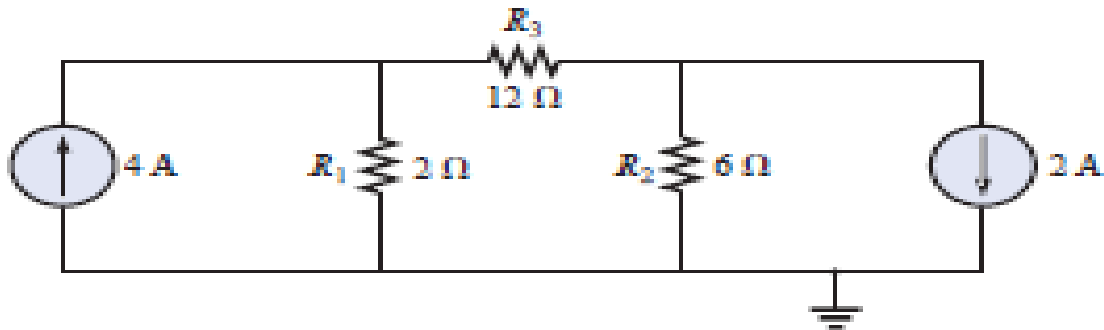
Introduction

We will now employ **Kirchhoff's Current Law (KCL)** to develop a method referred to as **nodal voltage method (nodal voltage method)**.

A **node** is defined as a junction of **two or more** branches (elements). If we now define one node of any network as a reference (that is, a point of zero potential or ground $V_{ref.}$), the remaining nodes of the network will all have a fixed potential relative to this reference. For a network of N nodes, therefore, there will exist $(N - 1)$ nodes with a fixed potential relative to the assigned reference node. Equations relating these nodal voltages can be written by applying Kirchhoff's current law at each of the $(N - 1)$ nodes. To obtain the complete solution of a network, these nodal voltages are then evaluated in the same manner in which loop currents were found in loop analysis.

- 1) Convert all the voltage sources in the circuit into the current sources.
- 2) Determine the number of nodes within the network.
- 3) Pick a reference node, and label each remaining node with a subscripted value of voltage: V_1 , V_2 , and so on.
- 4) Apply Kirchhoff's Current Law (KCL) at each node except the reference. Assume that all unknown currents leave the node for each application of Kirchhoff's current law. In other words, for each node, don't be influenced by the direction that an unknown current for another node may have had. Each node is to be treated as a separate entity, independent of the application of Kirchhoff's current law to the other nodes.
- 5) Solve the resulting equations for the nodal voltages (**No. of equations = $N-1$**).

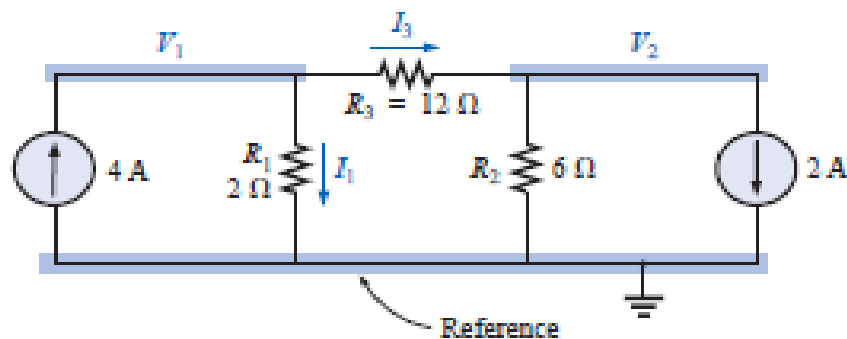
Example 1: For the circuit shown, Use **Nodal Voltage Method** to find the current through R_1 , R_2 and R_3



Solution:

There are 3 nodes (V_1 , V_2 , and $V_{ref.}$).

Therefore, two nodal voltages are required for V_1 and V_2



Applying KCL on the nodes,

For node1:

$$4 - I_1 - I_3 = 0 \rightarrow 4 - \frac{V_1}{R_1} - \frac{V_1 - V_2}{R_3} = 0$$

For node2:

$$I_3 - I_2 - 2 = 0 \rightarrow \frac{V_1 - V_2}{R_3} - \frac{V_2}{R_2} - 2 = 0$$

Re- arrange, then:

$$V_1 \left(\frac{1}{2} + \frac{1}{12} \right) - V_2 \left(\frac{1}{12} \right) = +4$$

$$V_2 \left(\frac{1}{12} + \frac{1}{6} \right) - V_1 \left(\frac{1}{12} \right) = -2$$

Note: The input currents to the node are indicated as (+ve), while the output currents are indicated as (-ve).

Note: The current sources have to transfer into the right side.

$$\frac{7}{12}V_1 - \frac{1}{12}V_2 = +4 \rightarrow 7V_1 - V_2 = 48$$

$$-\frac{1}{12}V_1 + \frac{3}{12}V_2 = -2 \rightarrow -1V_1 + 3V_2 = -24$$

$$7V_1 - V_2 = 48 \quad \dots (1)$$

$$-1V_1 + 3V_2 = -24 \quad \dots (2)$$

Solving by Determinates, then:

$$\Delta = \begin{vmatrix} 7 & -1 \\ -1 & 3 \end{vmatrix} = 20$$

$$\Delta_1 = \begin{vmatrix} 48 & -1 \\ -24 & 3 \end{vmatrix} = 120 \quad \text{and} \quad \Delta_2 = \begin{vmatrix} 7 & 48 \\ -1 & -24 \end{vmatrix} = -120$$

$$V_1 = \frac{\Delta_1}{\Delta} = \frac{\begin{vmatrix} 48 & -1 \\ -24 & 3 \end{vmatrix}}{\begin{vmatrix} 7 & -1 \\ -1 & 3 \end{vmatrix}} = \frac{120}{20} = 6V$$

$$V_2 = \frac{\Delta_2}{\Delta} = \frac{\begin{vmatrix} 7 & 48 \\ -1 & -24 \end{vmatrix}}{\begin{vmatrix} 7 & -1 \\ -1 & 3 \end{vmatrix}} = \frac{-120}{20} = -6V$$

Since V_1 is greater than V_2 , the assumed direction of I_3 is correct.

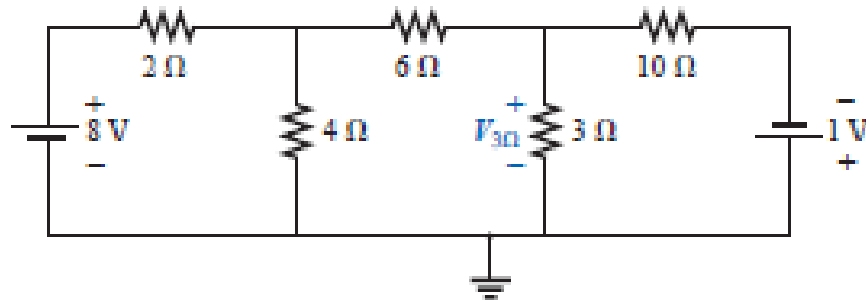
$$I_3 = \frac{V_1 - V_2}{R_3} = \frac{6 - (-6)}{12} = \frac{12}{12} = 1A$$

$$I_1 = \frac{V_1}{R_1} = \frac{6}{2} = 3A$$

$$I_2 = \frac{V_2}{R_2} = \frac{-6}{6} = -1A \quad (-ve)\text{sign means that the current in the circuit}$$

has the opposite direction.

Example2: For the circuit shown, Use Nodal Voltage method to find the voltage across 3Ω resistance and 6Ω . Then, find the current through 10Ω and 2Ω resistances.

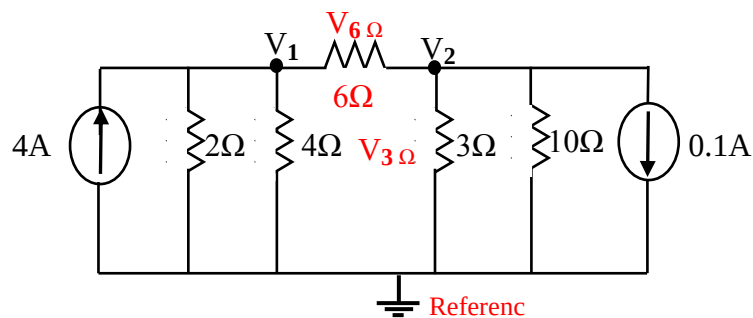
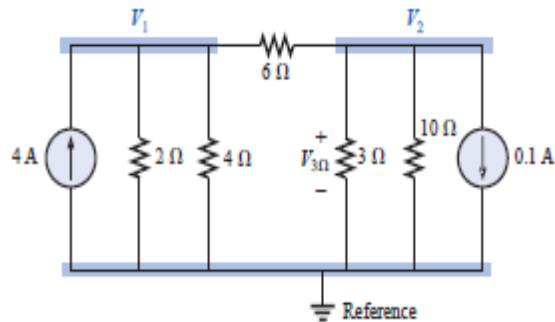


Solution

- Converting the voltage sources into the current sources and choosing nodes as shown.

Note: We have two nodes and reference node.

Therefore, two nodal voltages are required for V_1 and V_2



For node1:

$$\left(\frac{1}{2} + \frac{1}{4} + \frac{1}{6}\right)V_1 - \left(\frac{1}{6}\right)V_2 = 4 \rightarrow \left(\frac{11}{12}\right)V_1 - \left(\frac{1}{6}\right)V_2 = 4$$
$$11V_1 - 2V_2 = 48 \dots (1)$$

For node2:

$$\left(\frac{1}{10} + \frac{1}{3} + \frac{1}{6}\right)V_2 - \left(\frac{1}{6}\right)V_1 = -0.1 \rightarrow -\left(\frac{1}{6}\right)V_1 + \left(\frac{3}{5}\right)V_2 = -0.1$$
$$-5V_1 + 18V_2 = -3 \dots (2)$$

Re-arrange, then:

$$11V_1 - 2V_2 = 48 \dots (1)$$
$$-5V_1 + 18V_2 = -3 \dots (2)$$

$$\Delta = \begin{vmatrix} 11 & -2 \\ -5 & 18 \end{vmatrix} = 188$$

$$\Delta_1 = \begin{vmatrix} 48 & -2 \\ -3 & 18 \end{vmatrix} = 858 \quad \text{and} \quad \Delta_2 = \begin{vmatrix} 11 & 48 \\ -5 & -3 \end{vmatrix} = 207$$

$$V_2 = V_{3\Omega} = \frac{\Delta_2}{\Delta} = \frac{\begin{vmatrix} 11 & 48 \\ -5 & -3 \end{vmatrix}}{\begin{vmatrix} 11 & -2 \\ -5 & 18 \end{vmatrix}} = \frac{-33 + 240}{198 - 10} = 1.101V$$

$$V_1 = \frac{\Delta_1}{\Delta} = \frac{\begin{vmatrix} 11 & 48 \\ -5 & -24 \end{vmatrix}}{\begin{vmatrix} 11 & -2 \\ -5 & 18 \end{vmatrix}} = \frac{858}{188} = 4.56V$$

$$\therefore V_{6\Omega} = V_1 - V_2 = 4.56 - 1.101 = 3.459V$$

$$I_{10\Omega} = \frac{V_2}{R_{10\Omega}} = \frac{1.1}{10} = 0.11mA$$

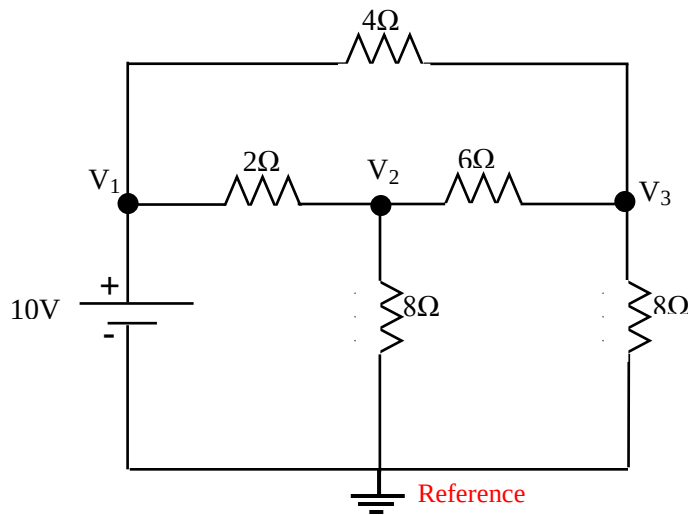
$$I_{2\Omega} = \frac{V_1}{R_{2\Omega}} = \frac{4.56}{2} = 2.28A$$

Special Cases: Nodal Analysis with Voltage Sources

Case1: When a voltage source is connected between the reference node and a non-reference node. This will simplify the analysis as shown.

Example: For the circuit shown:

- 1) Calculate the nodes voltage.
- 2) Find the current through 2Ω resistance.



In this circuit, we have 3 nodes voltages:

V_1 , V_2 and V_3

but $V_1 = 10V$

Thus, our analysis is somewhat amplified by this knowledge of the voltage at this node. We need only two nodal equations to analyze the circuit:

For node2:

$$\left(\frac{1}{8} + \frac{1}{2} + \frac{1}{6}\right)V_2 - \left(\frac{1}{6}\right)V_3 - \left(\frac{1}{2}\right)V_1 = 0 \quad \rightarrow 0.785V_2 - 0.16V_3 - 0.5V_1$$

For node3:

$$\left(\frac{1}{8} + \frac{1}{6} + \frac{1}{4}\right)V_3 - \left(\frac{1}{6}\right)V_2 - \left(\frac{1}{4}\right)V_1 = 0 \quad \rightarrow 0.535V_3 - 0.16V_2 - 0.25V_1$$

as $V_1 = 10V$

and so on.....Continue with same manner of solution....

Case2: (Super node) When a voltage source e is connected between (Two) non-reference nodes. In this case, a **(Super node)** is created.

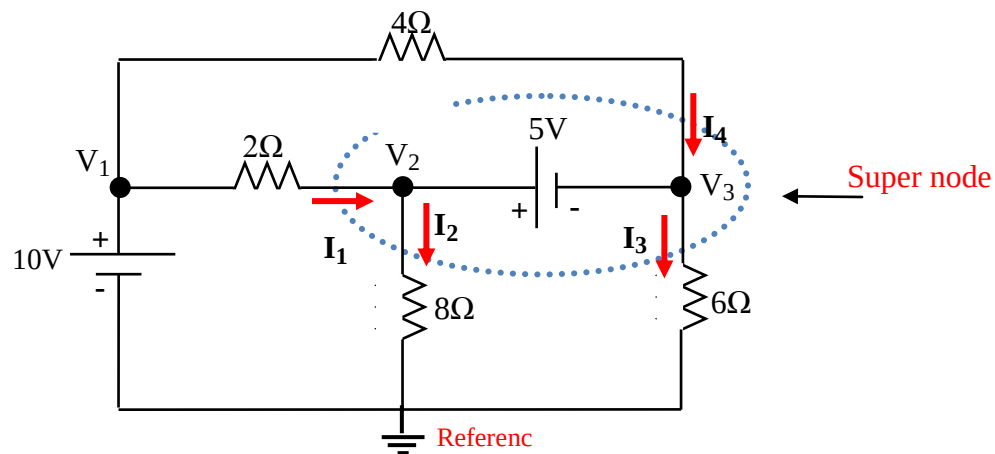
Example:

The voltage source (5V) is connected between **two** nodes (V_2 and V_3)

Therefore, V_2 and V_3 is called a **super node**

To solve the circuit, KCL and KVL must be applied on the super node:

Applying KVL at the super node results in:



Applying KCL at super node results in:

$$I_1 + I_4 = I_2 + I_3$$

$$\frac{V_1 - V_2}{2} + \frac{V_1 - V_3}{4} = \frac{V_2 - 0}{8} + \frac{V_3 - 0}{6}$$

$$V_1 = 10V$$

$$\frac{10 - V_2}{2} + \frac{10 - V_3}{4} = \frac{V_2}{8} + \frac{V_3}{6}$$

$$36 = 3V_2 + 2V_3$$

$$\text{We have: } 5 = V_2 - V_3$$

Solving for V_2 and V_3

$$V_2 = 9.2V$$

$$V_3 = 4.2V$$

H.W

Q) For the circuit shown, Use **Nodal voltage method**:

- 1) Calculate the voltage across 15Ω resistance.
- 2) Calculate the current through 2Ω resistance.
- 3) Find the power consumed by 4Ω resistance.

