
First Class

Circuit Theorems

1) Superposition Theorem

Introduction

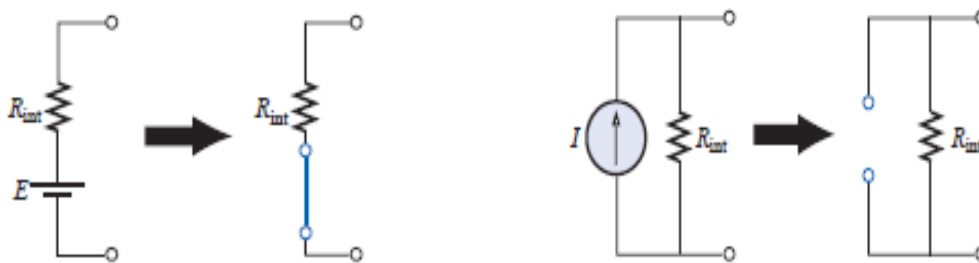
The **superposition theorem** can be used to find the solution to networks (circuits) with two or more sources that are not in series or parallel.

The superposition theorem states the following:

“The current through, or voltage across, an element in a linear bilateral network is equal to the algebraic sum of the currents or voltages produced independently by each source”.

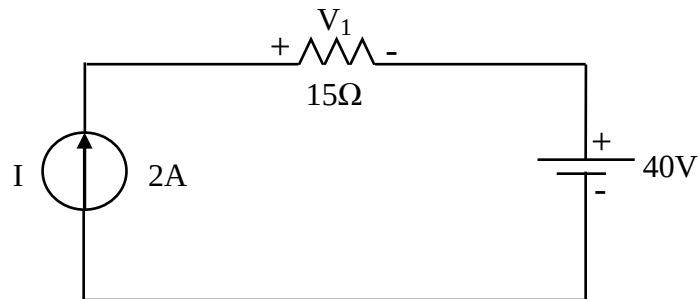
The most obvious advantage of this method is that it does not require the use of a mathematical technique such as determinants to find the required voltages or currents. Instead, each source is treated independently, and the algebraic sum is found to determine a particular unknown quantity of the network.

Removing the sources means **short circuit** the voltage source and **Open circuit** the current source.



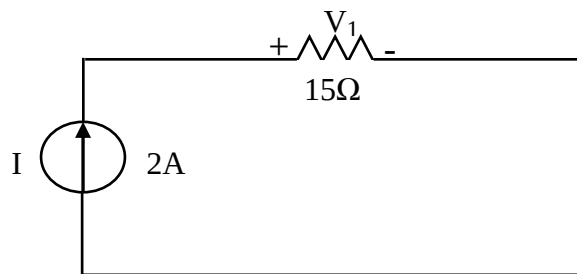
Removing the effect of the sources

Example1: Using the superposition theorem, determine V_1 for the circuit shown.



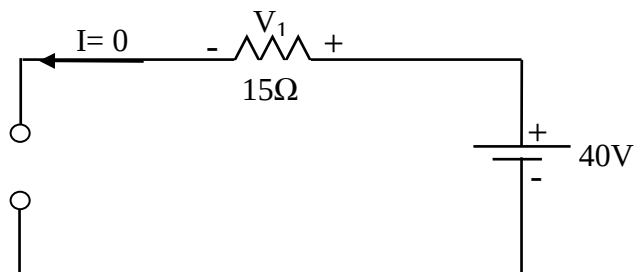
Effect Current Source:

$$\begin{aligned} V' &= IR_1 \\ &= (2)(15) = 30V \end{aligned}$$

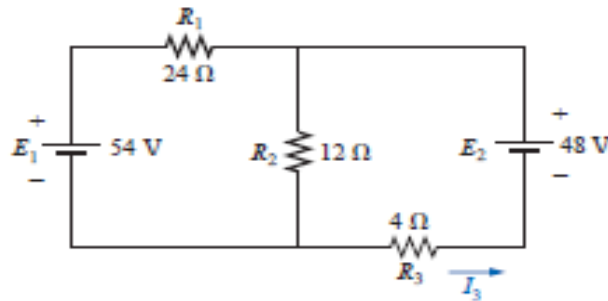


Effect voltage source:

$$\begin{aligned} V'' &= I_1 R_1 \\ &= (0)(15) = 0V \\ \therefore V_1 &= V' + V'' \\ &= 30 - 0 = 30V \end{aligned}$$



Example2: For the circuit shown, using the **superposition theorem**, determine the currents through 4Ω resistance.



Solution:

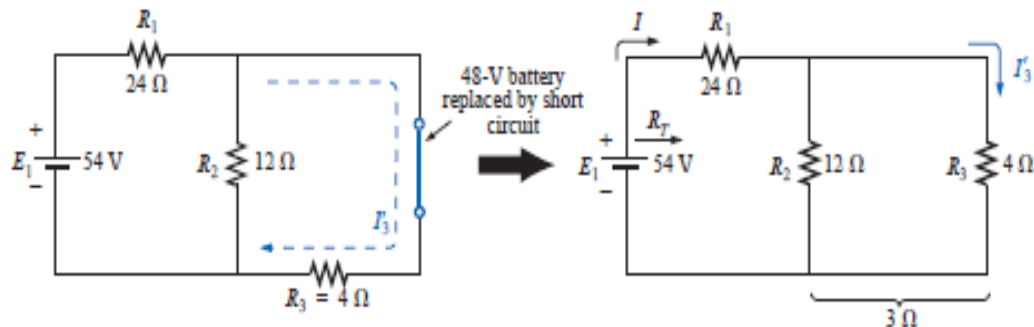
- The effect of E_1 :

$$I = \frac{E_1}{R_T} = \frac{54}{27} = 2A$$

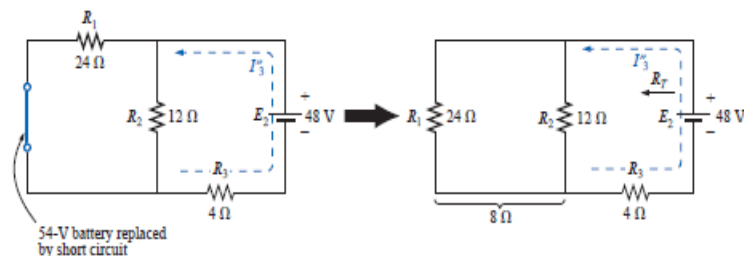
$$R_T = (R_2 // R_3) + R_1 \\ = (12 // 4) + 24 = 3 + 24 = 27\Omega$$

Using the current Divider rule

$$\therefore I_3' = I \frac{R_2}{R_2 + R_3} = 2 \frac{12}{12 + 4} = 1.5A$$



- The effect of E_2 :

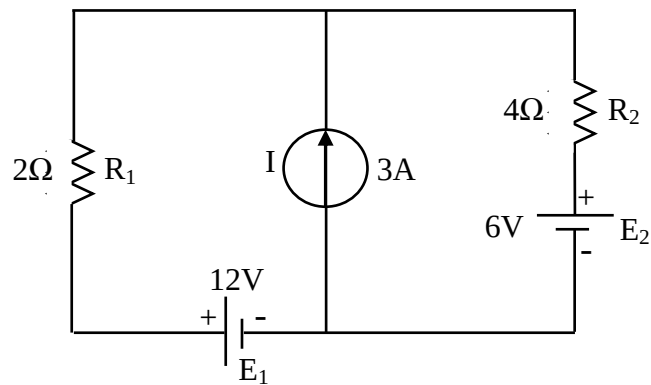


$$I = I_3'' = \frac{E_2}{R_T}$$

$$R_T = (24 // 12) + 4 = 8 + 4$$

$$\begin{aligned}
 &= 12\Omega \\
 \therefore I_3'' &= \frac{48}{12} = 4A \\
 \therefore I_3'' &= \frac{48}{12} = 4A \\
 \therefore I_3 &= I_3'' - I_3' \\
 I_3 &= 4 - 1.5 = 2.5A \text{ (in the direction of } I_3'')
 \end{aligned}$$

Example3: For the circuit shown, using the **superposition theorem**, determine the currents through 2Ω resistance.

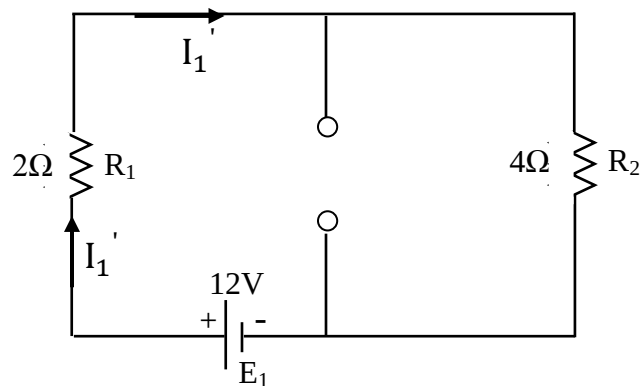


Solution:

- The effect of voltage source E_1

Remove the voltage source E_2 (short circuited) and the current source I (open circuit) as shown:

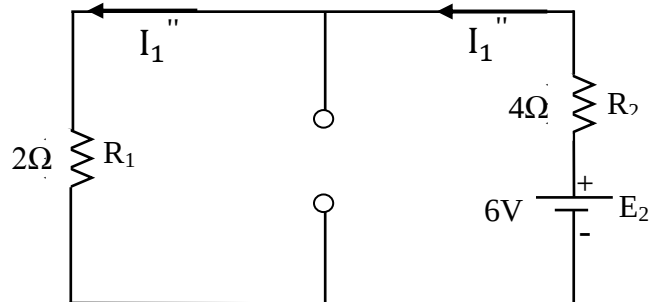
$$\therefore I_1' = \frac{E_1}{R_T} = \frac{12}{2 + 4} = 2A$$



- The effect of voltage source E_2

Remove the voltage source E_1 (short circuited) and the current source I (open circuit) as shown:

$$\therefore I_1'' = \frac{E_2}{R_T} = \frac{6}{2 + 4} = 1A$$

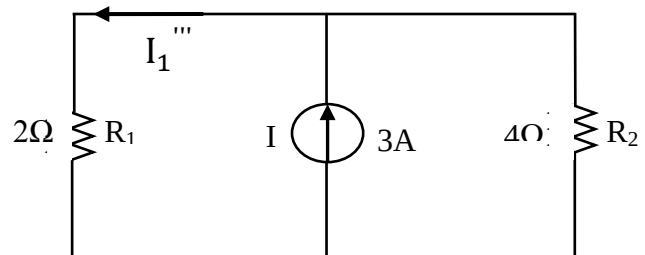


- The effect of current source I

Remove the voltage source E_1 and E_2 (short circuited) as shown:

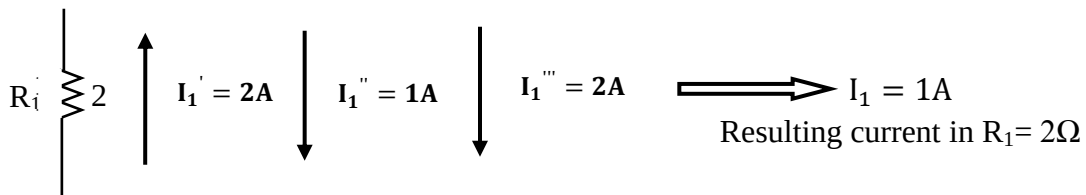
Using the current Divider rule

$$\therefore I_1''' = I \frac{R_2}{R_1 + R_2} = (3) \frac{4}{4 + 2} = 2A$$



$$\therefore I_1 = I_1'' + I_1''' - I_1' \rightarrow I_1 = 1 + 2 - 2 = 1A$$

same direction Opposite direction



H.W

Q.) For the circuit shown, using **Superposition Theorem**:

- 1) Calculate the current through 4Ω resistor.
- 2) Find the voltage across 4Ω .

