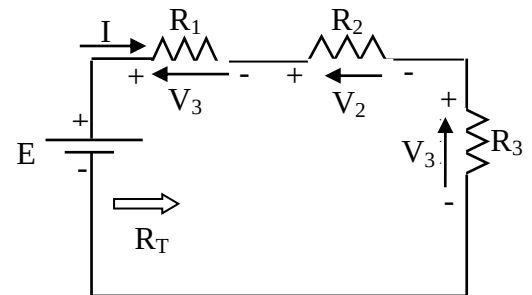

First Class/ Lecture Two

Circuit Transformations

Series Circuits: Two elements are in series if they have only one point in common that is not connected to other current carrying elements of the network.

For the series circuit shown, using KVL, we have:

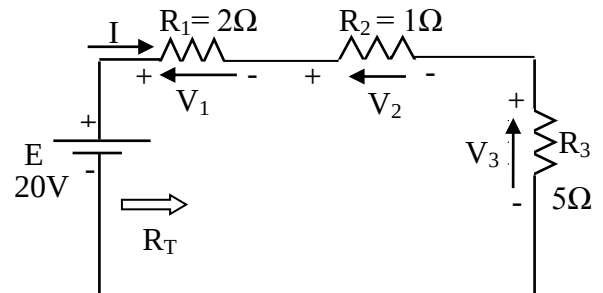
$$\begin{aligned} E &= V_1 + V_2 + V_3 \\ &= IR_1 + IR_2 + IR_3 = I(R_1 + R_2 + R_3) \\ &= IR_T \quad \text{as } R_T = (R_1 + R_2 + R_3) \\ \therefore I &= \frac{E}{R_T} \end{aligned}$$



In general. For a series circuit consisting N resistors, then the total resistance of such circuit, R_T is given as: $R_T = R_1 + R_2 + R_3 + \dots R_N$

Example: For the circuit shown;

- 1) Find the total resistance
- 2) Calculate the current I
- 3) Determine the voltages V_1 , V_2 and V_3



Solution: 1) $R_T = R_1 + R_2 + R_3$
 $= 2 + 1 + 5 = 8\Omega$

2) $I = \frac{E}{R_T} = \frac{20}{8} = 2.5A$

3) $V_1 = IR_1 = (2.5)(2) = 5V$

$V_2 = IR_2 = (2.5)(1) = 2.5V$

$V_3 = IR_3 = (2.5)(5) = 12.5V$

Note: $V_1 + V_2 + V_3 = E$

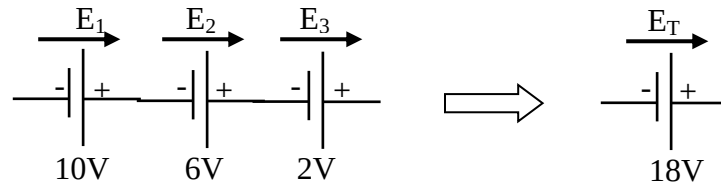
$5 + 2.5 + 12.5 = 20V$

$\therefore 20V = 20V$

Voltage Sources in Series:

The net voltages will be the algebraic sum of all sources that are connected in series.

Example:



$$\begin{aligned}E_T &= E_1 + E_2 + E_3 \\&= 10 + 6 + 2 = 18V\end{aligned}$$

Voltage Divider Rule: Consider the series circuit shown

We have: $R_T = R_1 + R_2$

$$\text{and } I = \frac{E}{R_T}$$

$$V_1 = IR_1 = \left(\frac{E}{R_T}\right) \times R_1$$

$$= \frac{E \cdot R_1}{R_T}$$

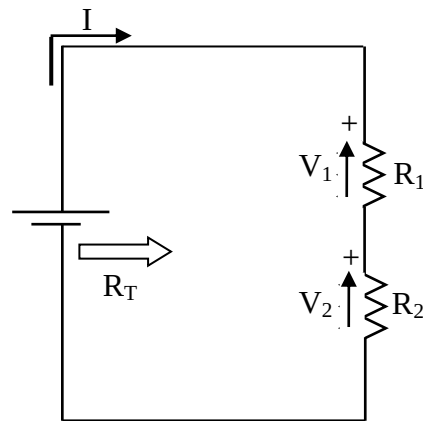
Similarly

$$V_2 = IR_2 = \left(\frac{E}{R_T}\right) \times R_2$$

$$= \frac{E \cdot R_2}{R_T}$$

Hence, we can write;

$$V_X = \frac{E \cdot R_X}{R_T} \quad \text{Voltage divider rule}$$



This means that the voltage divider rule can be understood to state that:

The voltage across a resistor in a series circuit is equal to the value of that resistor times the total applied voltage across the series elements divided by the total resistance of the series elements.

Example: Determine the voltages V_1 , V_3 and V' for the circuit shown;

Solution: Using the voltage divider rule:

$$V_1 = \frac{E \cdot R_1}{R_T}$$
$$= \frac{(45) \cdot (2 \times 10^3)}{(2 + 5 + 8) \times 10^3} = 6V$$

$$V_3 = \frac{E \cdot R_3}{R_T}$$
$$= \frac{(45) \cdot (8 \times 10^3)}{(2 + 5 + 8) \times 10^3} = 24V$$

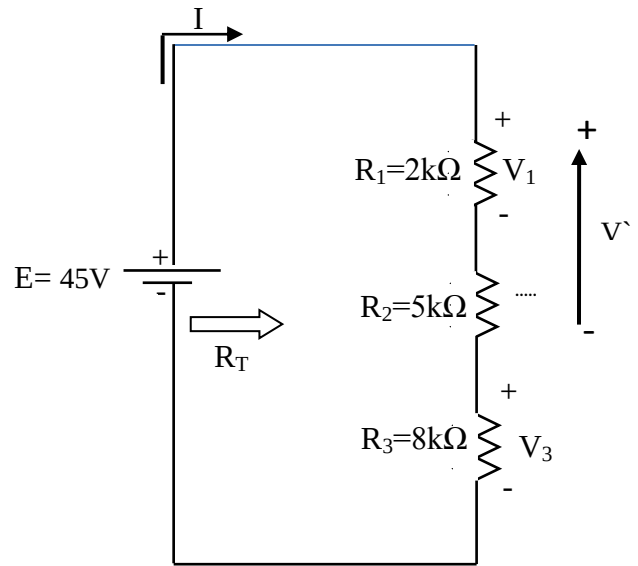
$$V' = \frac{E \cdot R'}{R_T} \text{ Where } R' = (2 + 5) \times 10^3 = 7 \times 10^3 \Omega$$

$$= \frac{(45) \cdot (7 \times 10^3)}{(2 + 5 + 8) \times 10^3} = 21V$$

Note that $E = V_3 + V'$

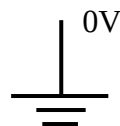
$$45 = 24 + 21$$

$$45V = 45V$$



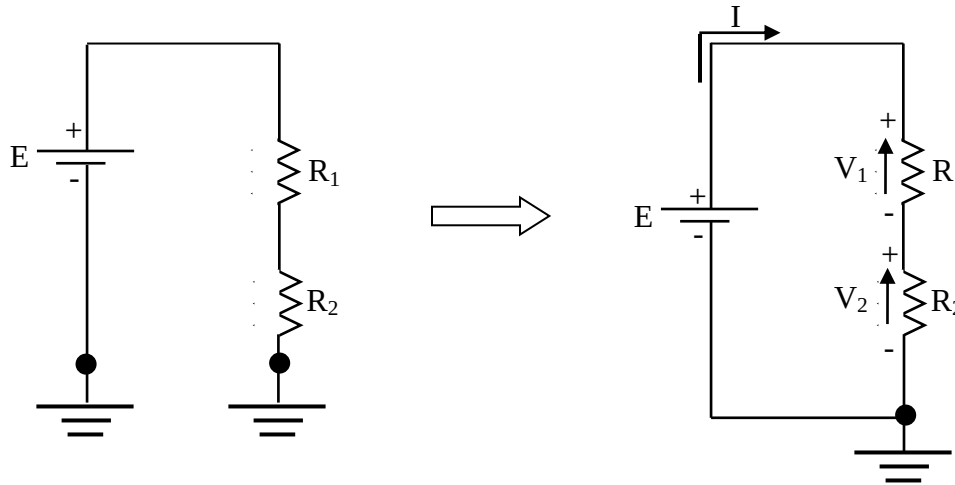
Ground Potential

It is common, for safety purposes and as a reference to ground electrical and electronic systems. The symbol for the ground connection is:

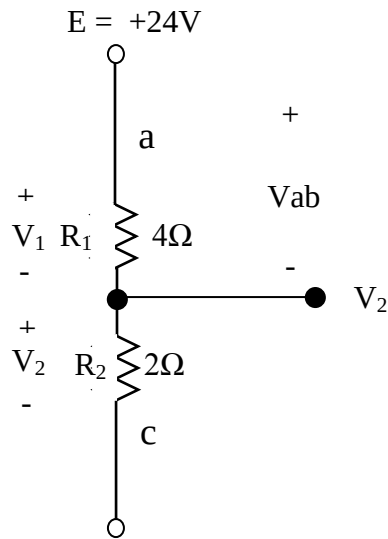


With its defined potential level (Zero volts). As a consequence, the circuit might need to be redrawn in the ordinary form so as to be analyzed.

Example: For the circuit shown; with ground potential connected for the source and to elements; this circuit can be redrawn to make it easy analyzing it.

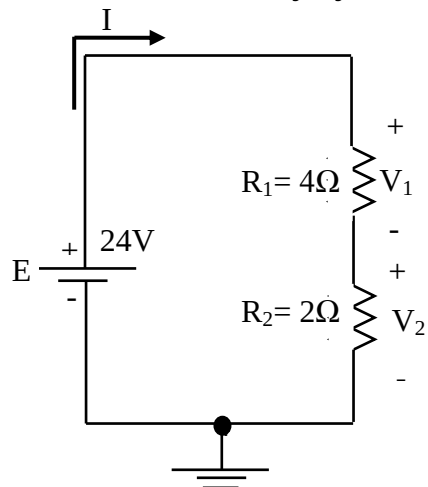


Example: Using the voltage divider rule, determine the voltages V_1 and V_2 for the circuit shown:



Solution:

Redrawing the circuit, with the standard battery symbol then the circuit will be as shown below.



$$\therefore V_1 = \frac{E \cdot R_1}{R_T} = \frac{24 \times 4}{4 + 2} = 16V$$

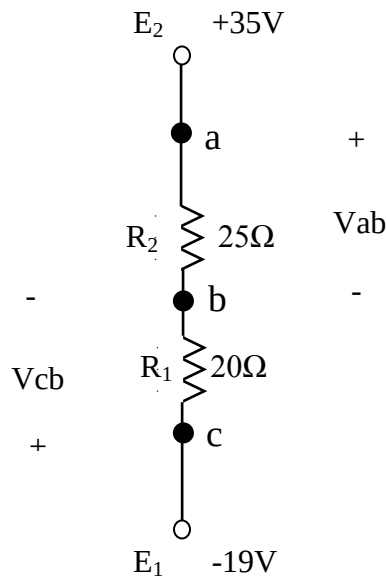
$$\text{and } V_2 = \frac{E \cdot R_2}{R_T} = \frac{24 \times 2}{4 + 2} = 8V$$

$$E = V_1 + V_2$$

$$E = 16 + 8$$

$$24V = 24V$$

Example: Using the voltage divider rule, determine the voltages V_{ab} and V_{cb} and V_b for the circuit shown:



Solution: The circuit redrawn as shown:

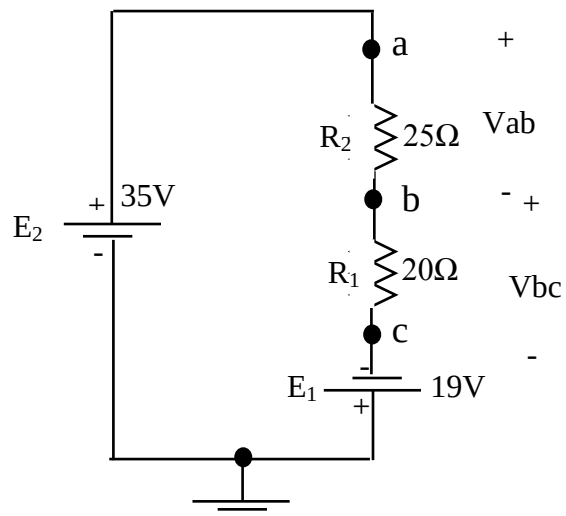
$$I = \frac{E_1 + E_2}{R_T} = \frac{19 + 35}{20 + 25} = 1.2A$$

$$V_{ab} = IR_2 = (1.2)(25) = 30V$$

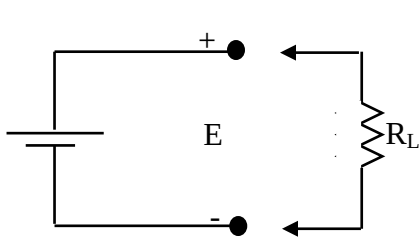
$$\text{and } V_{cb} = -V_{bc} = -V_{bc}$$

$$= -IR_1 = -(1.2)(20) = -24V$$

$$V_c = -E_1 = -19V$$

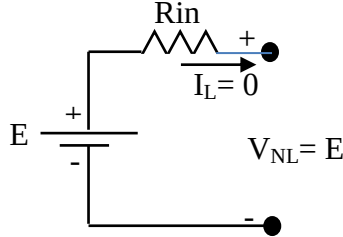


Internal Resistance of Voltage Sources:



Ideal voltage source

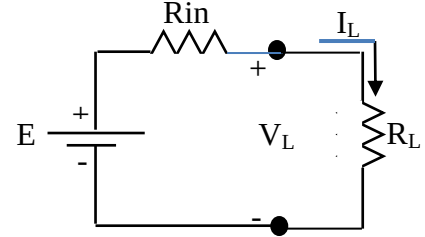
(1)



Actual voltage source

(at no load)

(2)



Actual source
(Connected to load R_L)

(3)

- Applying KVL on Figure (3): $E - I_L R_{in} - V_L = 0$
 - Applying KVL on Figure (2): $E = V_{NL}$
- Substituting, then: $V_{NL} - I_L R_{in} - V_L = 0$

$$\therefore V_L = V_{NL} - I_L R_{in}$$

Parallel Circuits:

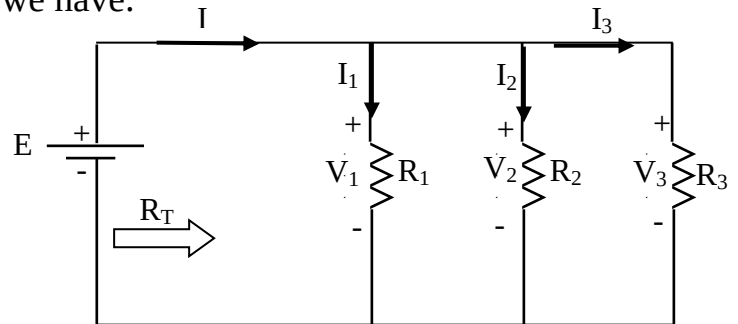
Two branches or elements or networks are in parallel if they have two points in common.

For the parallel circuits shown; using KCL, we have:

$$\begin{aligned} I &= I_1 + I_2 + I_3 \\ &= \frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3} \end{aligned}$$

Since $V_1 = V_2 = V_3 = E = V$

$$\begin{aligned} \therefore I &= \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3} \\ \therefore \frac{V}{R_T} &= V \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \right) \\ \therefore \frac{1}{R_T} &= \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \right) \end{aligned}$$



In general, for N resistors connected in parallel, then:

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots + \frac{1}{R_N}$$

Notation: Conductance (G)

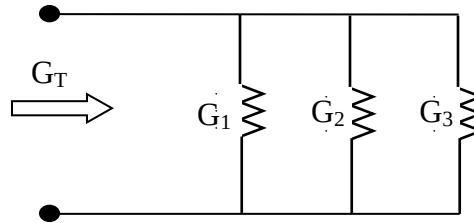
For parallel networks, it is common to use the idea of conductance in the circuit analysis. The conductance (G) is defined as:

$$G = \frac{1}{R} \quad \text{Siemens (S)}$$

So, we can write the total conductance G_T for the parallel circuit shown as;

$$G_T = G_1 + G_2 + G_3$$

$$\Rightarrow R_T = \frac{1}{G_T}$$



Note that, the total resistance R_T of parallel resistors is always less than the value of the smallest resistor.

Special Cases:

The general relation for the total resistance of parallel resistors as:

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots + \frac{1}{R_N}$$

Case1: For equal resistors in parallel, ie when:

$$R_1 = R_2 = R_3 \dots \dots R_N = R$$

Then;

$$\frac{1}{R_T} = \frac{1}{R} + \frac{1}{R} + \frac{1}{R} + \dots + \frac{1}{R}$$

$$\therefore \frac{1}{R_T} = N \left(\frac{1}{R} \right)$$

$$\therefore R_T = \frac{R}{N}$$

Case2: For Two resistors in parallel, then R_T is given as;

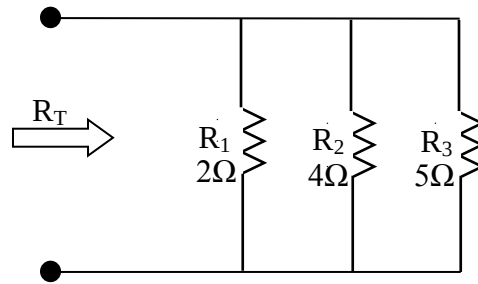
$$R_T = \frac{R_1 R_2}{R_1 + R_2}$$

This means, that the total resistors of Two parallel resistors is the product of the Two divided by their sum.

Case3: For Three resistors in parallel, then R_T is given as;

$$R_T = \frac{R_1 R_2 R_3}{R_1 R_2 + R_1 R_3 + R_2 R_3}$$

Example: Determine the total resistance for the circuit shown:



Solution:

$$\begin{aligned}\frac{1}{R_T} &= \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \\ \frac{1}{R_T} &= \frac{1}{2} + \frac{1}{4} + \frac{1}{5} \\ &= 0.5 + 0.25 + 0.2 = 0.95 \\ \therefore R_T &= \frac{1}{0.95} = 1.053\Omega\end{aligned}$$

Example: For the parallel network shown;

- 1) Calculate R_T
- 2) Determine I_T
- 3) Calculate I_1 and I_2
- 4) Determine the power to each resistive load
- 5) Determine the power delivered by the source and compare it with the dissipated by the resistive elements.

Solution:

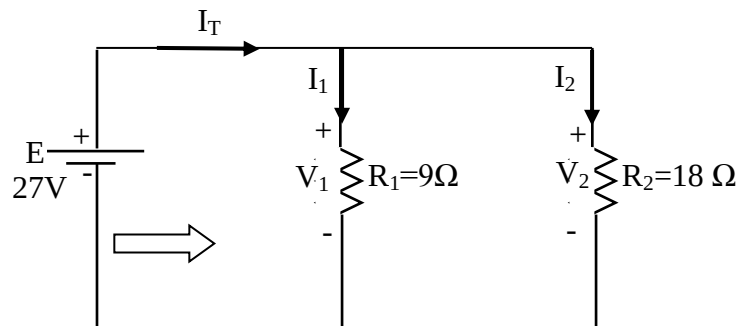
1)

$$R_T = \frac{R_1 R_2}{R_1 + R_2} = \frac{(9)(18)}{9 + 18} = \frac{162}{27} = 6\Omega$$

$$I_T = \frac{E}{R_T} = \frac{27}{6} = 4.5A$$

$$I_1 = \frac{V_1}{R_1} = \frac{E}{R_1} = \frac{27}{9} = 3A$$

$$I_2 = \frac{V_2}{R_2} = \frac{E}{R_2} = \frac{27}{18} = 1.5A$$



Note that

$$I_T = I_1 + I_2$$

$$P_1 = I_1 V_1 = E I_1 = (27)(3) = 81W$$

$$P_2 = I_2 V_2 = E I_2 = (27)(1.5) = 40.5W$$

$$P_s = E I_T = (27)(4.5) = 121.5W$$

Or $P_s = P_1 + P_2 = 81 + 40.5 = 121.5W$

Note that the power dissipated by the resistors R_1 and R_2 equals to the power delivered by the source E .

Current Divider Rule:

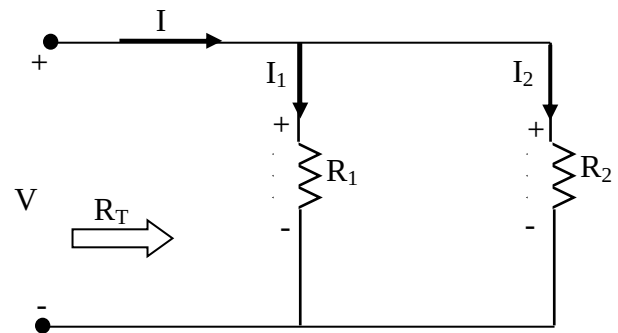
Consider the parallel circuit shown; we have;

$$I_2 = \frac{V}{R_T} \Rightarrow V = I R_T$$

$$R_T = \frac{R_1 R_2}{R_1 + R_2}$$

$$\text{but } I_1 = \frac{V}{R_1} = \frac{I R_T}{R_1} = I \frac{R_1 R_2}{R_1 + R_2} \frac{1}{R_1}$$

$$\therefore I_1 = I \times \frac{R_2}{R_1 + R_2}$$

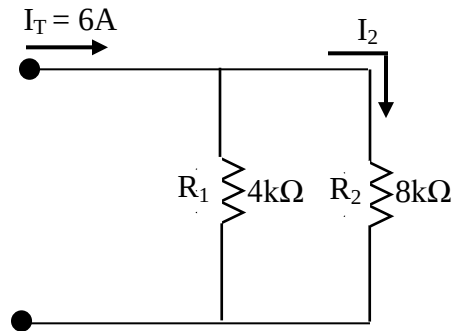


Similarly, for I_2 , we have:

$$I_2 = \frac{V}{R_2} = \frac{I R_T}{R_2} = I \frac{R_1 R_2}{R_1 + R_2} \frac{1}{R_2}$$

$$\therefore I_2 = I \times \frac{R_1}{R_1 + R_2}$$

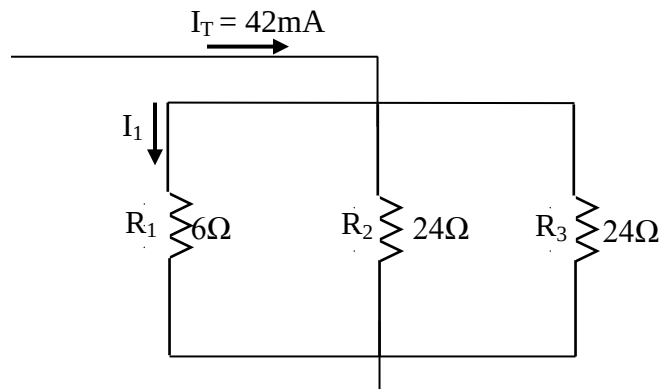
Example: Determine the current I_2 for the network shown:



Solution:

$$I_2 = I_T \frac{R_1}{R_1 + R_2}$$
$$= 6 \times \frac{4 \times 10^3}{(4 + 8) \times 10^3} = 6 \times \frac{4}{4 + 8} = 2A$$

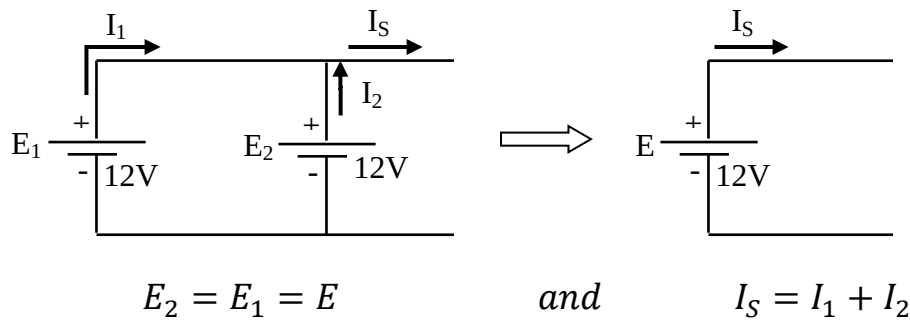
Example: Find the current I_1 for the network shown:



Solution:

$$I_1 = I_T \frac{R_T}{R_1}$$
$$R_T = \frac{R_1 R_2 R_3}{R_1 R_2 + R_1 R_3 + R_2 R_3}$$
$$\text{Or } \frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} = \frac{1}{6} + \frac{1}{24} + \frac{1}{24}$$
$$\therefore R_T = 4\Omega$$
$$\therefore I_1 = 42 \times 10^{-3} \frac{4}{6} = 28 \times 10^{-3} = 28mA$$

Voltage Sources in Parallel: To increase the current rating of the sources, **two or more** batteries in parallel of the same terminal voltage would be used.

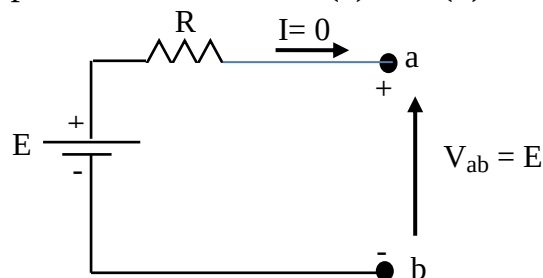


Open and short Circuits:

We often need to apply open and short circuits in the analysis for electric networks.

Open Circuit: An open circuit is simply two isolated terminals not connected by an element of any kind.

Consider the circuit shown with open circuit terminals (a) and (b).



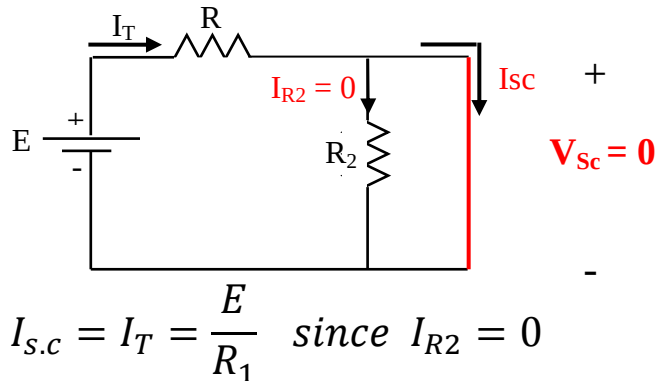
$$V_{\text{open circuit}} = V_{\text{oc}} = V_{ab} = E$$

Since I in the open circuit = 0

In general: An open circuit can have a potential difference (voltage) across its terminals but the current is always **ZERO**

Short Circuit: An open circuit is direct connection of zero ohms across an element or a combination of elements.

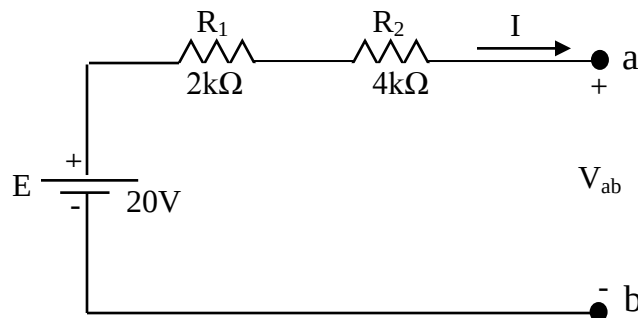
Consider the circuit shown with short circuit across the resistor R_2 .



$$V_{short\ circuit} = V_{s.c} = 0$$

In general: A short circuit can carry a current of any level, but the potential difference (voltage) across its terminals is always **ZERO**.

Example: For the network shown, determine V_{ab} .

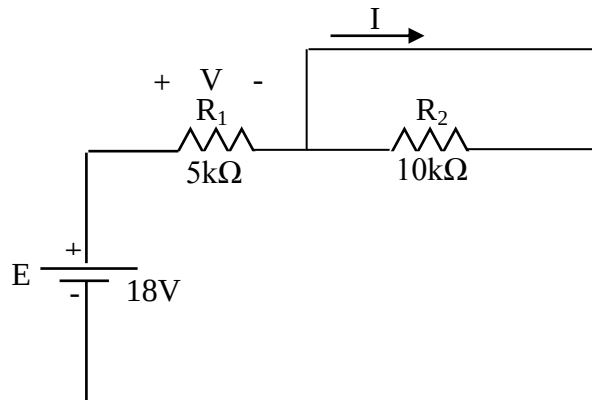


Solution: We have an open circuit across the terminal (a) and (b), the

$$I = 0 \Rightarrow V_1 = 0 \text{ and } V_2 = 0$$

$$\text{Applying KVL} \Rightarrow V_{ab} = E = 20V$$

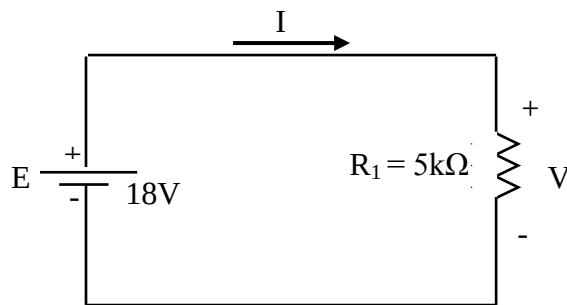
Example: Calculate I and V for the circuit shown;



When we have a short circuit across R_2

Therefore, **No current through R_2**

The circuit can be redrawn to be as shown;



$$I = \frac{E}{R_1} = \frac{E}{R_1 + 0} = \frac{18}{5k\Omega} = 3.6mA$$

$$\therefore V = IR_1 = E = 18V$$