

Series- Parallel Circuits:

For the circuit shown,

$$R_1 = R_A = 4 \Omega$$

$$R_B = R_2 // R_3 = \frac{R_N}{N} = \frac{4}{2} = 2 \Omega$$

$$R_C = R_4 + R_5 = 0.5 + 1.5 = 2 \Omega$$

$$R_B // R_C = \frac{R_N}{N} = \frac{2}{2} = 1 \Omega$$

From Figure 2:

$$R_T = R_A + R_B // R_C = 4 + 1 = 5 \Omega$$

$$\therefore I_T = \frac{E}{R_T} = \frac{10}{5} = 2A$$

Also, From Figure 2:

$$I_B = I_C = \frac{I_A}{2} = \frac{2}{2} = 1A$$

From Figure 1:

$$I_{R4} = I_{R5} = I_C = 1A$$

From Figure 1:

$$V_A = I_A R_A = (2)(4) = 8V$$

$$V_B = I_B R_B = (1)(2) = 2V$$

$$V_C = I_C R_C = (1)(2) = 2V$$

For check by KVL

$$\sum V = 0 \Rightarrow E - V_A - V_B = 0$$

$$10V = 8 + 2$$

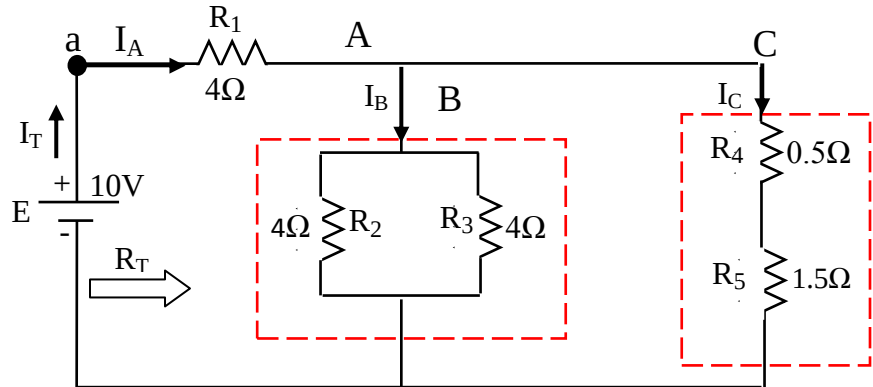


Figure (1)

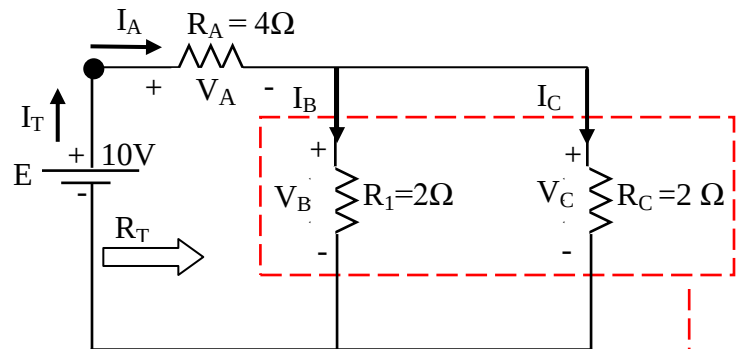


Figure (2)

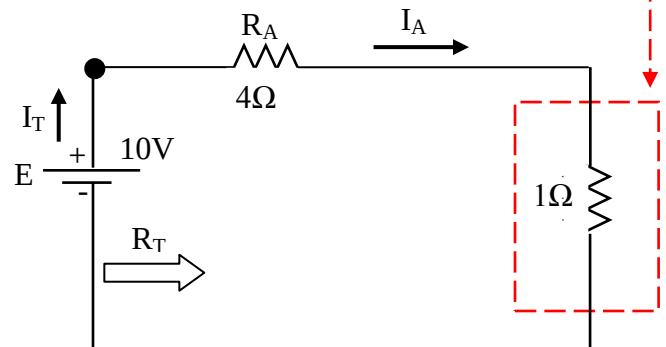
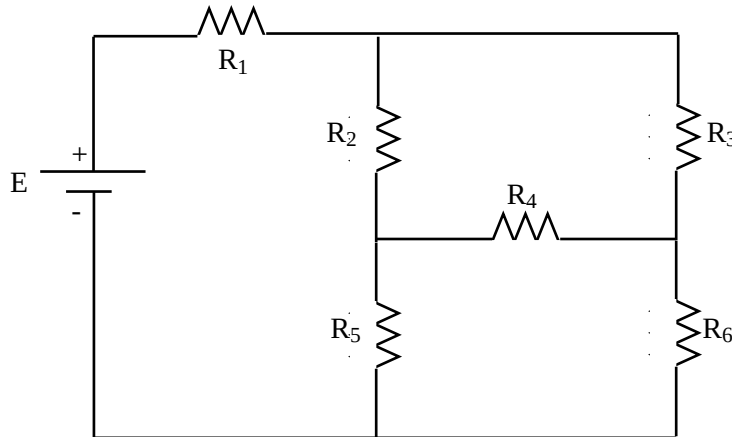


Figure (3)

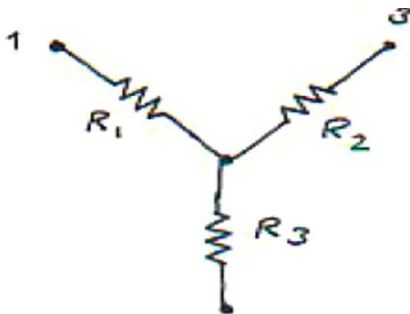
Wye (Y) - Delta (Δ) Transformation

There are some cases often arises in the circuit analysis when the resistors are **neither in parallel nor in series**. For example, consider the circuit shown:



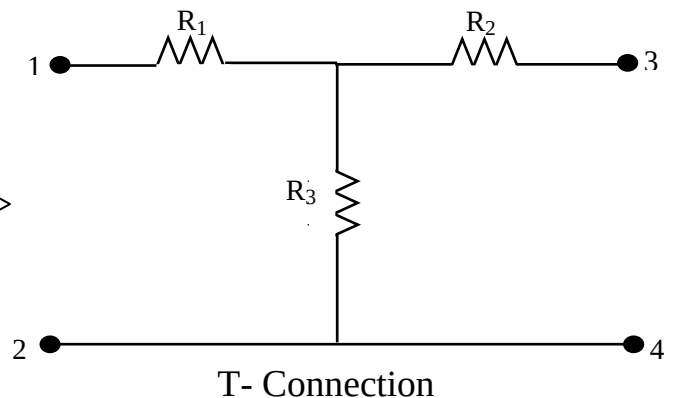
In this circuit, R_1 , R_2 , R_3 R_6 are neither in series nor in parallel.

Wye (Y or Star or T) connection:



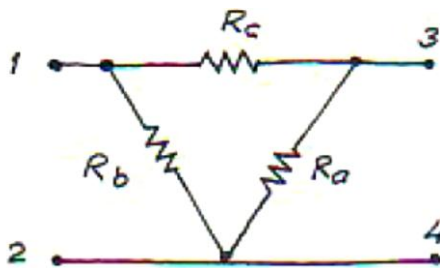
Y or Star circuit connection

Equivalent



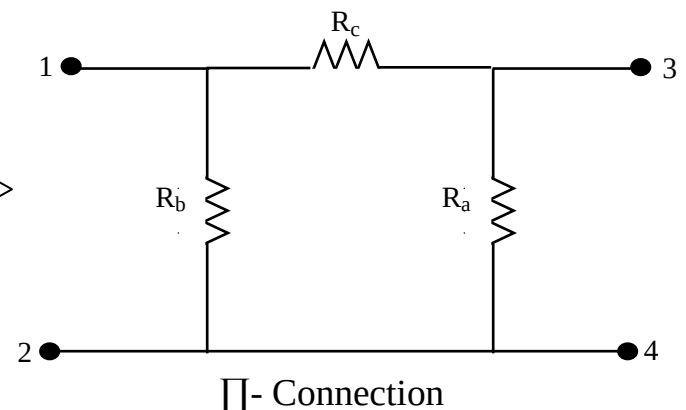
Y and T connection

Delta (Δ or $\square$) connection:



$\square$ or Delta circuit connection

Equivalent

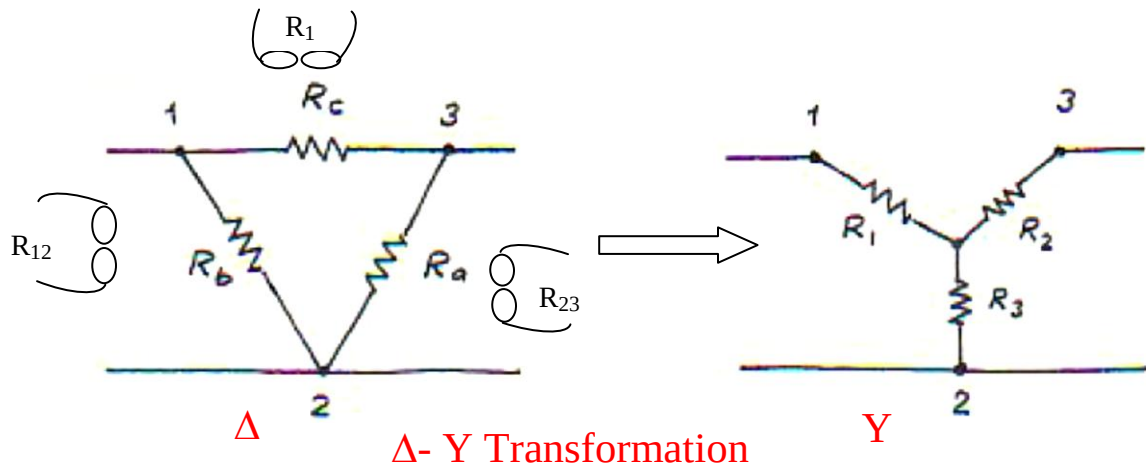


$\square$ - Connection

Delta circuit connection and its equivalent $\square$ connection

Delta (Δ) – Star (Y) Transformation:

- We have Δ and want to get its equivalent Y circuit.
- Consider the Δ circuit shown; to be transformed into its equivalent Y shown below:



$$R_{12} = R_b // (R_a + R_c) = \frac{R_b (R_a + R_c)}{R_a + R_b + R_c}$$

$$R_{13} = R_c // (R_a + R_b) = \frac{R_c (R_a + R_b)}{R_a + R_b + R_c}$$

$$R_{23} = R_a // (R_b + R_c) = \frac{R_a (R_b + R_c)}{R_a + R_b + R_c}$$

$$R_{12} = R_1 + R_3$$

$$R_{13} = R_1 + R_2$$

$$R_{23} = R_2 + R_3$$

$$\therefore R_1 + R_3 = \frac{R_b (R_a + R_c)}{R_a + R_b + R_c} \dots \dots (1)$$

$$R_1 + R_2 = \frac{R_c (R_a + R_b)}{R_a + R_b + R_c} \dots \dots (2)$$

$$R_2 + R_3 = \frac{R_a (R_b + R_c)}{R_a + R_b + R_c} \dots \dots (3)$$

Subtraction Eq. (3) from Eq. (1) and **adding** the resulting equation to Eq. (1) result in:

$$R_1 = \frac{R_b \times R_c}{R_a + R_b + R_c}$$

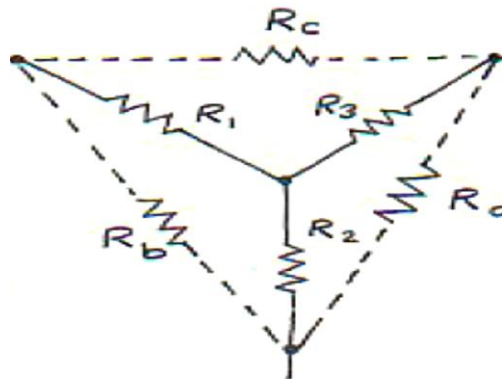
$$R_2 = \frac{R_c \times R_a}{R_a + R_b + R_c}$$

$$R_3 = \frac{R_a \times R_b}{R_a + R_b + R_c}$$

In general: Each resistor in the required **Y** network is equal the product of two adjacent resistors of **Δ** resistors divided by the sum of all **Δ** resistors.

Wye (Y) – Delta (Δ) Transformation:

- We have **Y** and want to get its equivalent **Δ** circuit.
- Consider the **Y** circuit shown; to be transformed into its equivalent **Δ** shown below:



$$R_a = \frac{R_1 R_2 + R_1 R_3 + R_2 R_3}{R_1}$$

$$R_b = \frac{R_1 R_2 + R_1 R_3 + R_2 R_3}{R_3}$$

$$R_c = \frac{R_1 R_2 + R_1 R_3 + R_2 R_3}{R_2}$$

In general: Each resistor in the required **Δ** network is the sum of all possible products of **Y** resistors taken two at a time, divided by the opposite **Y** resistor.

Note:

$$R_1 = R_2 = R_3 = R_Y$$

$$\text{and } R_a = R_b = R_c = R_\Delta$$

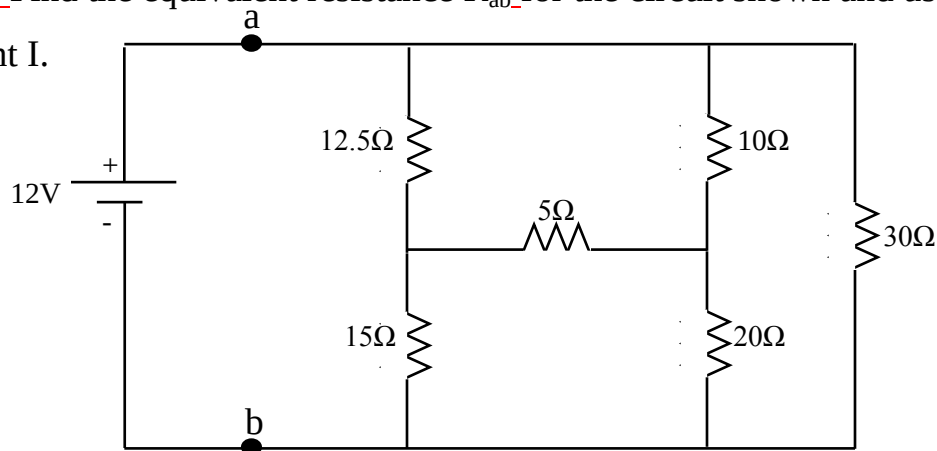
- Under Balanced consideration, the conversion equations becomes:

$$R_Y = \frac{R_\Delta}{3}$$

or

$$R_\Delta = 3R_Y$$

Example: Find the equivalent resistance R_{ab} for the circuit shown and use it to find the current I.



Solution:

We can't use the relations of series connected or parallel connected resistors to find R_{ab} .

Therefore, we try to use Δ - Y transformations or Y- Δ to Find R_{ab} .

If we transform the Y consisting of;

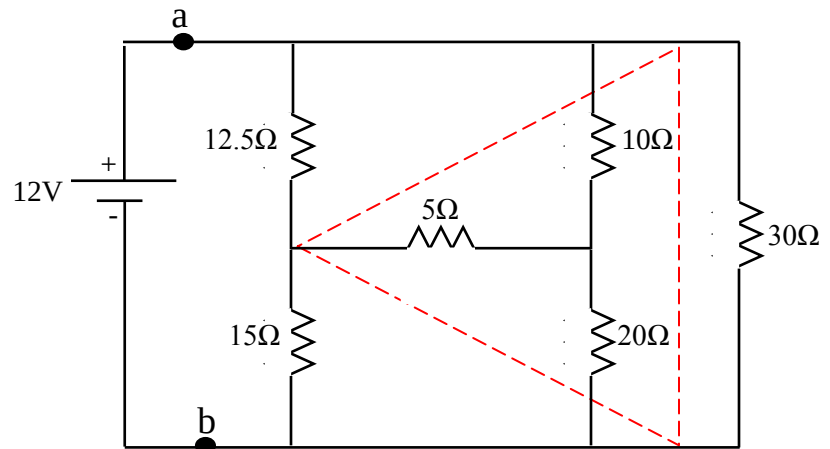
$$R_1 = 10\Omega$$

$$R_2 = 20\Omega$$

$$R_3 = 5\Omega$$

$\therefore$ The equivalent Δ circuit contains:

$$R_a = \frac{R_1 R_2 + R_1 R_3 + R_2 R_3}{R_1}$$



$$= \frac{10 \times 20 + 20 \times 5 + 5 \times 10}{10} = \frac{350}{10}$$

$$= 35\Omega$$

Similarly;

$$R_b = \frac{R_1 R_2 + R_1 R_3 + R_2 R_3}{R_3} = \frac{350}{20} = 17.5\Omega$$

$$\text{and } R_c = \frac{R_1 R_2 + R_1 R_3 + R_2 R_3}{R_2} = \frac{350}{5} = 70\Omega$$

$$12.5 // 17.5 = \frac{12.5 \times 17.5}{12.5 + 17.5} = 7.3\Omega$$

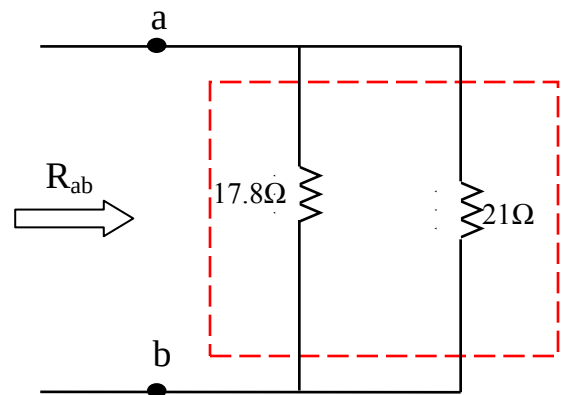
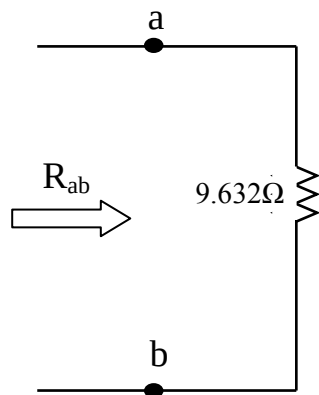
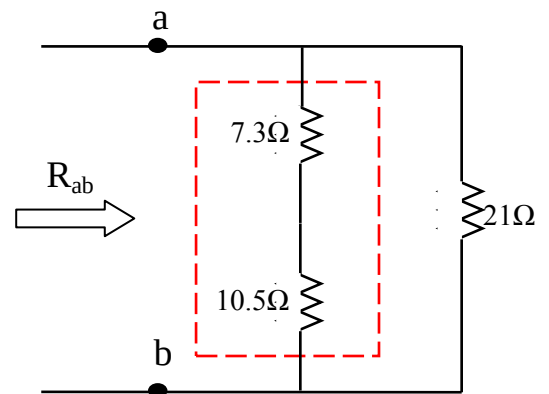
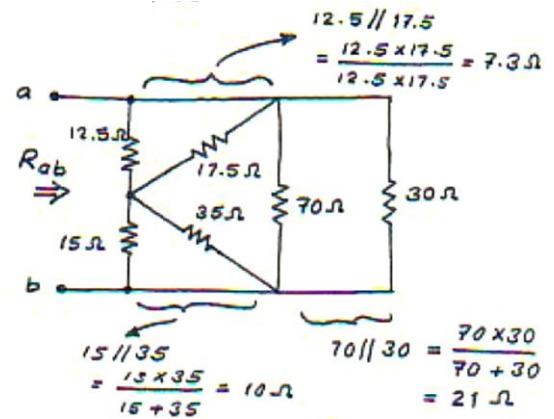
$$15 // 35 = \frac{15 \times 35}{15 + 35} = 10.5\Omega$$

$$70 // 30 = \frac{70 \times 30}{70 + 30} = 21\Omega$$

$$\therefore R_{ab} = (7.3 + 10.5) // 21 = \frac{17.8 \times 21}{17.8 + 21}$$

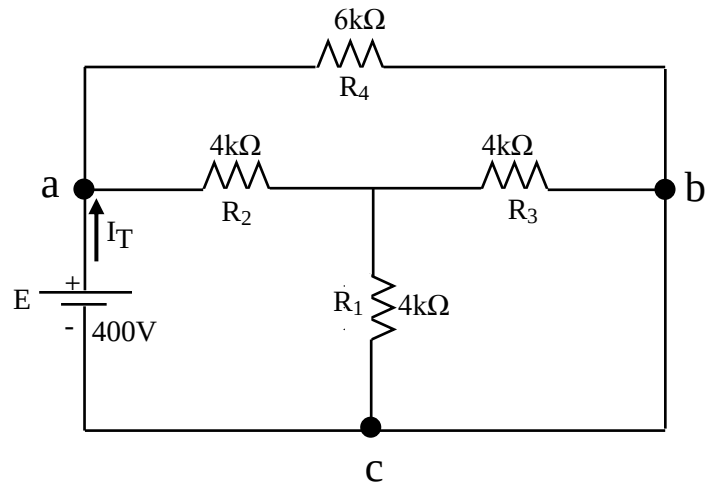
$$R_{ab} = 9.632\Omega$$

$$\therefore I = \frac{E}{R_{eq}} = \frac{E}{R_{ab}} = \frac{12}{9.632} = 1.246A$$



Example: For the circuit shown,

- 1) Calculate Req.
- 2) Find the current I



Solution:

There is no series or parallel circuit elements to be simplified.

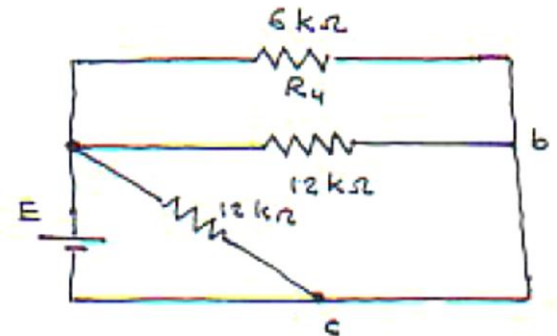
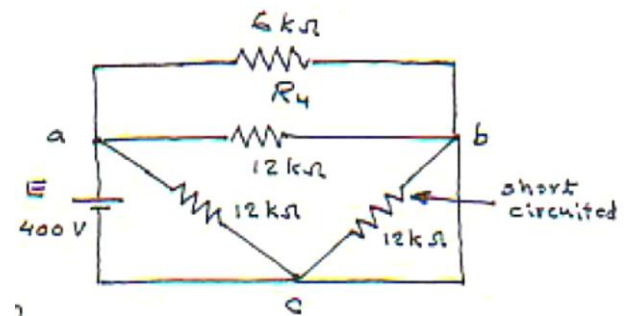
Convert the **Y** labeled (abc) to **Δ**.

$$R_a = \frac{R_1 R_2 + R_1 R_3 + R_2 R_3}{R_1}$$

Since $R_1 = R_2 = R_3 = 4k\Omega$

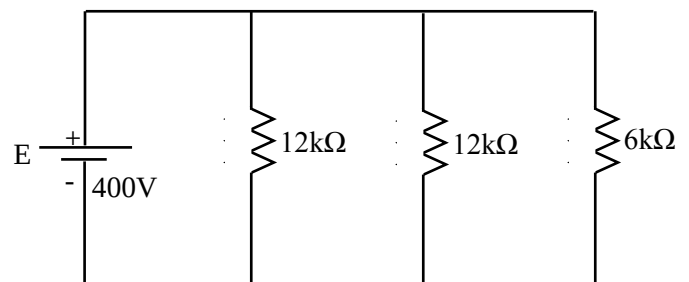
$$\therefore R_a = 3R_1 = 3(4k\Omega) = 12k\Omega$$

and $R_a = R_b = R_c = 12k\Omega$



$$R_T = 12k\Omega // 12k\Omega // 6k\Omega = 3k\Omega$$

$$I_T = \frac{E}{R_T} = \frac{400}{3 \times 10^3} = 133.33mA$$



Sources Transformation

Introduction

It is often necessary or convenient to have a voltage source rather than a current source or a current source rather a voltage source.

In the circuit shown, we have a voltage source connected to load resistance R_L :

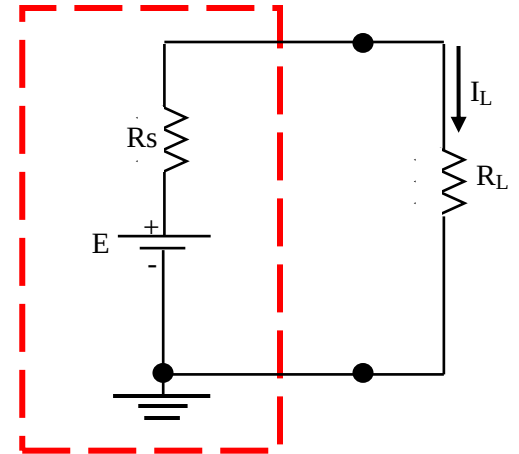
$$\text{We have } I_L = \frac{E}{R_T} = \frac{E}{R_s + R_L}$$

Multiplying the numerator by $R_s/R_s = 1$, we have

$$I_L = \frac{\left(R_s/R_s\right)E}{R_s + R_L} = \frac{R_s(E/R_s)}{R_s + R_L}$$

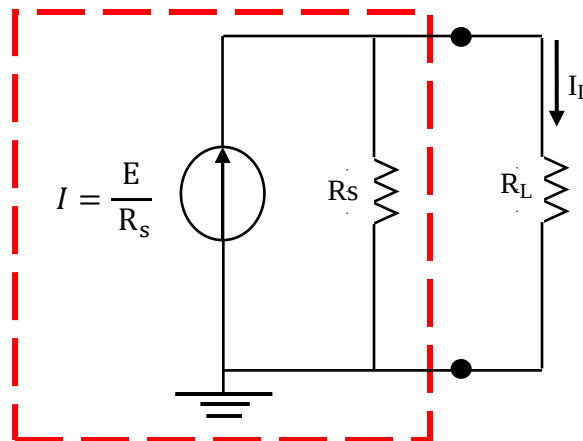
$$\text{Since } E/R_s = I$$

$$\therefore I_L = \frac{R_s \times I}{R_s + R_L}$$



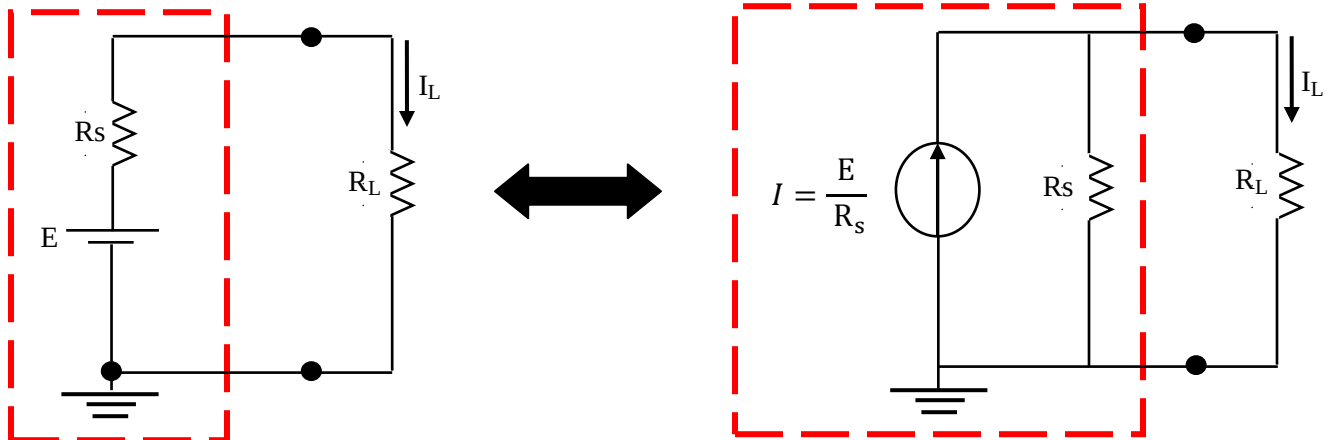
Practical voltage source

This is current divider equation, which can be represented by the circuit below, which is the equivalent circuit of the voltage source.



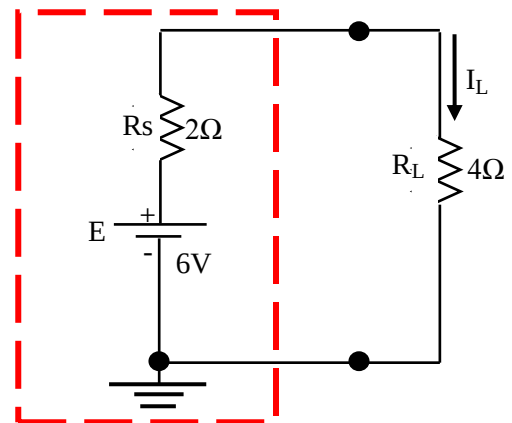
Practical current source

In general, a voltage source with voltage E and series resistance R_s can be replaced by a current source with a current I and parallel resistor R_s as shown:

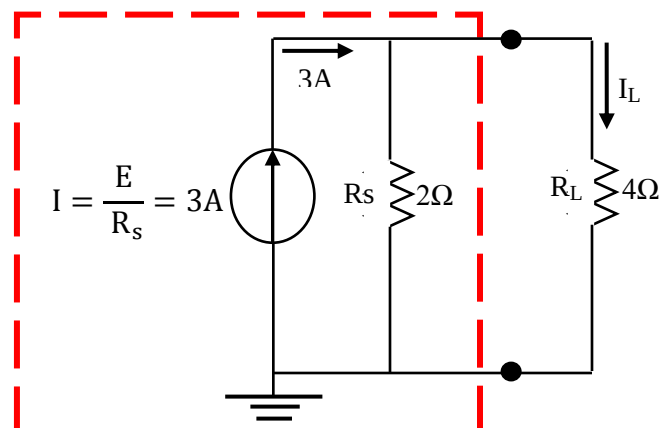


Example: Convert the voltage source in the circuit shown into a current source, and calculate the current through the load (R_L) for each source.

$$I_L = \frac{E}{R_s + R_L} = \frac{6}{2 + 4} = 1A$$



$$I_L = \frac{I \times R_s}{R_s + R_L} = \frac{(3)(2)}{2 + 4} = 1A$$



Current source

Voltage source

$$I_L = \frac{I R_s}{R_s + R_L} = \frac{(9 \times 10^{-3})(3 \times 10^3)}{(3 + 6) \times 10^3} = 3 \times 10^{-3} = 3\text{mA}$$
$$I_L = \frac{E}{R_T} = \frac{E}{R_s + R_L} = \frac{27}{(3 + 6) \times 10^3} = 3 \times 10^{-3} = 3\text{mA}$$

H.W:

Q.) Convert the current source in the circuit shown into a voltage source, and calculate the current through 6Ω resistance.

