

WORKING FLUID

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The term fluid includes the liquid, vapor and gas phases of the pure substance. It is defined as the quantity of mater contained within the boundaries of the system. The thermodynamic properties of the working fluid are analyzed and studied with the help of property diagrams such as P-V, P-T, T-S,..... etc. The state of the working fluid is represented as a point on any property diagram as shown in fig (3.1).

PHASES of WORKING FLUID

As mentioned earlier that there three phases for the working fluid namely **solid, liquid and gas** phase. In thermodynamics we concerned mostly with the liquid and gas phases. The vapor is a transition phase between the liquid and the gas phases. Little attention is devoted to the solid phase.

In general the liquid phase has almost constant specific volume independent of pressure and temperature and takes the shape of its container. When the liquid is heated at constant pressure its temperature rises significantly while its specific volume remains almost constant until bubbles begin to form in the liquid. This phenomena is known as **boiling process**. As the pressure at which heat is supplied increases the boiling occurs at higher temperature. The relation between boiling pressure and boiling temperature is shown in fig (3.2). Line ABC is the saturated liquid line.

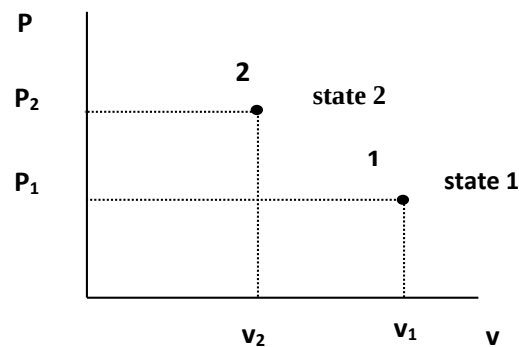


Fig 3.1

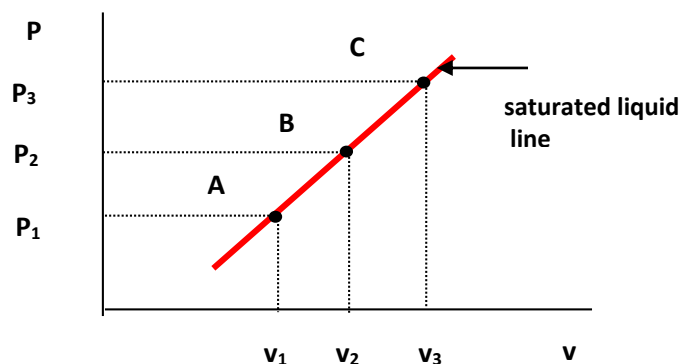


Fig 3.2

The change of phase starts with beginning of boiling process and finishes at the end of boiling process where all the liquid changes to vapor. During this process the pressure and temperature remains constant as mentioned previously. The heat supplied during the boiling process is known as **latent heat of evaporation**.

At the end of boiling (evaporation) process the working fluid is known as saturated vapor. When the pressure is changed the temperature at which the boiling process ends is also changed and hence a line similar to saturated liquid line is obtained and known as the saturated vapor line. The two lines meet at a point known as the critical point as shown in fig (3.3).

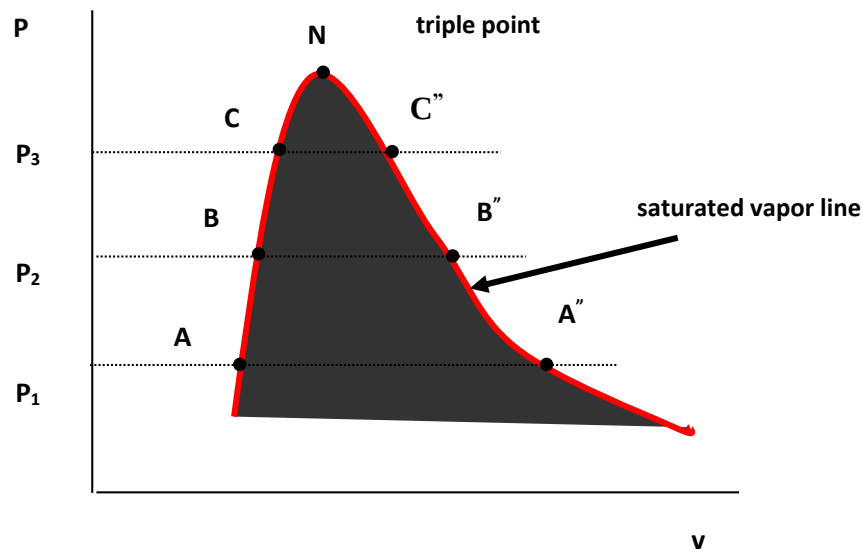


Fig (3.3)

The region to the left of the saturated liquid line is the liquid phase region, the region to the right of the saturated vapor line is the vapor phase region and the shaded area is the mixed phase region.

When different heating processes are performed at different constant temperatures ie, the heat is supplied at different constant temperature different constant temperature lines are obtained on the P-V diagram known as isothermal lines or isotherms as shown in fig (3.4).

When the saturated vapor is heated further its temperature and specific volume increase significantly and it becomes superheated vapor. The superheated vapor is the vapor whose temperature is higher than the saturation temperature at vapor pressure. The difference between the vapor actual temperature and the saturation temperature at its pressure is known as the **degree of superheat** which is a property of superheated vapor.

There are three terms related to the mixed phase region must clarified which are the wet vapor, the dryness fraction and the wetness fraction.

The wet vapor is the vapor that composed of a mixture of saturated vapor and fine droplets of saturated liquid.

The dryness fraction is the mass of saturated vapor in 1 kg of wet vapor which is given the symbol (x). For wet vapor ($0 < x < 1$) and, for saturated vapor ($x=1$) and for saturated liquid ($x=0$).

The wetness fraction is the mass of saturated liquid droplets in 1 kg of wet vapor ($1-x$).

As mentioned earlier the vapor is a transition phase between the liquid phase and gas phase. As the degree of superheat of the vapor increases it approaches the behavior of the perfect gas and when the isotherm becomes a hyperbola the vapor turns into perfect gas and the relation between its pressure and volume at constant temperature can be described by the following equation,

$$(Pv)_T = C \quad (3-1)$$

This means that it follows the equation of state of perfect gas.

$$\frac{Pv}{T} = C \quad \text{where} \quad (3.2)$$

P =pressure (Pa), v =specific volume (m^3/kg), T =absolute temperature (K) and C =constant.

When the working fluid becomes a perfect gas its behavior is governed by specific mathematical relationships between its properties known as gas laws. However there are no such laws that govern the behavior of the working fluid in the liquid phase and vapor. There are tables based on experimental data used to obtain the properties of the liquid and vapor at any state. The most common of these tables is the steam tables which is used for water vapor. It must mentioned that there are similar tables for vapors of other pure substances such as ammonia vapor tables and refrigerant vapor tables.

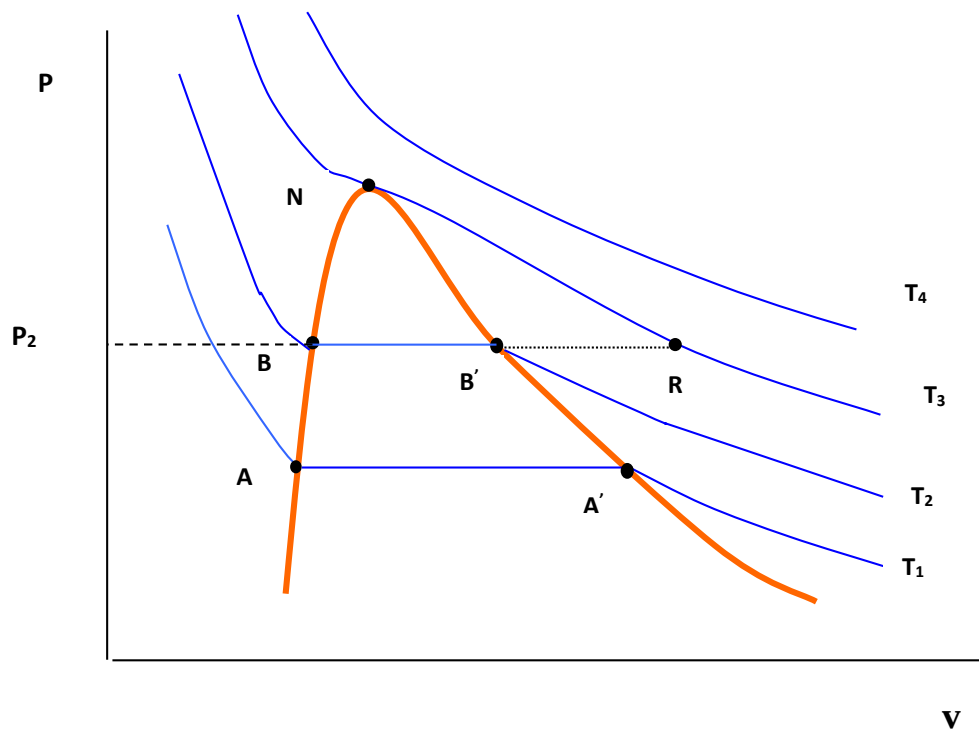


Fig (3.4) Isothermal Lines (isotherms)

HOW TO USE STEAM TABLES

Steam tables are classified into two parts, saturated steam table and superheated steam table.

These tables are given in terms of pressure and temperature. Let us start with the saturated steam table and their use.

SATURATED STEAM TABLE

There are two tables for saturated steam one in terms of pressure and the other in terms of temperature but the data in both tables are approximately the same.

The properties given in these tables are the properties of saturated liquid in addition to the properties of saturated vapor, which include:

specific volume (v) m^3/kg

specific internal energy (u) kJ/kg

specific enthalpy (h) kJ/kg

specific entropy (s) $kJ/kg.K$

saturation pressure (P) Pa , kPa , MPa

saturation temperature (t_s) $^{\circ}C$

The properties of saturated liquid are given the subscript (f) such as (v_f, u_f, h_f, s_f) while the properties of saturated vapor are given the subscript (g) such as (v_g, u_g, h_g, s_g). The subscript (fg) refers to the difference between the properties of saturated vapor and saturated liquid such as (u_{fg}, h_{fg}, s_{fg}).

$$u_{fg} = u_g - u_f \quad (3-3)$$

$$h_{fg} = h_g - h_f \quad (3-4)$$

$$s_{fg} = s_g - s_f \quad (3-5)$$

Table (3-1) A representative part of saturated steam table

<i>P</i>	<i>t</i>	<i>v_f</i>	<i>v_g</i>	<i>u_f</i>	<i>u_g</i>	<i>h_f</i>	<i>h_{fg}</i>	<i>h_g</i>	<i>s_f</i>	<i>s_g</i>
Mpa	°C	m³/kg	m³/kg	kJ/kg	kJ/kg	kJ/kg	kJ/kg	kJ/kg	kJ/kg.K	kJ/kg.K
1.0	179.9	.00113	.1943	761.5	2582	763	2013.6	2776.2	2.138	6.583
10	311	.00145	.01804	1393	2547	1408	1319.7	2727.5	3.361	5.62
20	365.7	.00204	.00588	1786	2301	1827	592	2418.4	4.015	4.941

Table (3-1) shows a small section of the saturated steam table. The following remarks are noticed in this table.

1-The specific volume of saturated liquid is very small compared with that of the saturated vapor at low pressures but this difference decreases at high pressures until they are equal at the critical point.

2-The specific internal energy, specific enthalpy and specific entropy of saturated liquid are zero at the triple point.

3-The latent heat of evaporation is zero at the critical point.

4-The values of saturated liquid properties rises as the pressure increases.

The saturated vapor table is given either in terms of saturated temperature or in terms of saturated pressure from the triple point until the critical point.

The properties of wet vapor are obtained from the saturated vapor table with the help of specific mathematical relations based on the fact that the wet vapor is a mixture of saturated liquid and saturated vapor. For example the specific volume of a wet vapor equals the sum of the specific volume of saturated liquid fraction plus the specific volume of the saturated vapor fraction, ie.

$$v = (1 - x)v_f + xv_g \quad (3-6)$$

The term $(1 - x)v_f$ is the specific volume of saturated liquid and the term xv_g is the specific volume of saturated vapor. Since the specific volume of saturated liquid is very small it is neglected in most cases.

Similarly,

$$h = (1 - x)h_f + xh_g \quad (3-7)$$

$$h = h_f + x(h_g - h_f) \quad (3-8)$$

$$h = h_f + xh_{fg} \quad (3-9)$$

$$h_{fg} = h_g - h_f$$

$$u = (1-x)u_f + xu_g \quad (3-10)$$

$$u = u_f + x(u_g - u_f) \quad (3-11)$$

$$s = (1-x)s_f + xs_g \quad (3-12)$$

$$s = s_f + x(s_g - s_f) \quad (3-13)$$

As mentioned earlier the change of phase occurs at constant pressure and constant temperature. Therefore it is represented as a horizontal line on the P-V diagram. By applying the following energy equation which will be studied in more details later the latent heat of evaporation can be calculated.

$$q = (u_2 - u_1) + w \quad (3-14)$$

State (1) represents the start of evaporation (saturated liquid (*f*)) and state (2) represents the end of evaporation (saturated vapor (*g*)). The symbol (*w*) represents the work done during the process while the symbol (*q*) represents the heat supplied during the process.

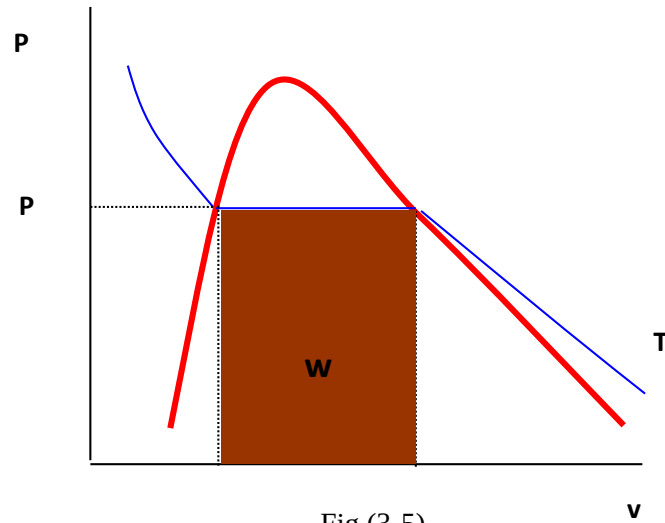


Fig (3-5)

The work done is represented by the shaded area on fig (3-5). Therefore, since the pressure is constant during change of phase,

$$w = P(v_2 - v_1) \quad (3-15)$$

$$\therefore q = (u_g - u_f) + P(v_g - v_f) \quad (3-16)$$

$$q = (u_g + Pv_g) - (u_f + Pv_f) \quad (3-17)$$

$$h = u + Pv \quad (3-17)$$

$$\therefore h_f = u_f + Pv_f \quad (3-18)$$

$$h_g = u_g + Pv_g \quad (3-19)$$

$$q = h_g - h_f \quad (3-20)$$

$$\therefore q = h_{fg} \quad (3-21)$$

Example (3-1)

Calculate the dryness fraction, specific volume and specific internal energy for steam at a pressure of 0.7 MPa with a specific enthalpy of 2600 kJ/kg.

Solution procedure

The first step in solving vapor problems is to check the quality of vapor in order to choose the appropriate table.

Now at this pressure (0.7 MPa) we find from the saturated steam table that the specific enthalpy (h_g) of saturated steam equals 2764 kJ/kg which larger than the enthalpy given in the example. Hence this steam is wet and we must find its dryness fraction.

The required properties of saturated liquid and saturated steam at this pressure are:

$$h_f = 697 \text{ kJ/kg} , h_{fg} = 2067 \text{ kJ/kg}$$

$$v_f = 0.001108 \text{ m}^3/\text{kg} , v_g = 0.273 \text{ m}^3/\text{kg}$$

$$u_f = 696.3 \text{ kJ/kg} , u_g = 2571.1 \text{ kJ/kg}$$

$$\therefore h = h_f + xh_{fg}$$

$$x = \frac{h - h_f}{h_{fg}} = \frac{2600 - 697}{2067} = 0.921$$

$$v = (1 - x)v_f + xv_g$$

$$\therefore v = (1 - 0.921) \times 0.001108 + 0.921 \times 0.273 = 0.2515205 \text{ m}^3/\text{kg}$$

$$u = u_f + x(u_g - u_f)$$

$$\therefore u = 696.3 + 0.921 \times (2571.1 - 696.3) = 2422.99 \text{ kJ/kg}$$

SUPERHEATED VAPOR TABLE

Since the pressure and temperature in the superheated vapor region are independent properties the superheated steam table is arranged in terms of pressure and temperature. The properties given in this table include: pressure, temperature, specific volume or density, specific internal energy, specific enthalpy and specific entropy. The data is given through a wide range of pressure and temperature.

Fig (3.6) A Sample of Superheated Steam Table

T °C	v m ³ /kg	u kJ/kg	h kJ/kg	s kJ/(kg·K)	v m ³ /kg	u kJ/kg	h kJ/kg	s kJ/(kg·K)	v m ³ /kg
P = 0.01 MPa (45.81°C)*									
50	14.674	2437.8	2584.7	8.1502	3.240	2483.9	2645.9	7.5906	1.054
50	14.889	2443.9	2592.8	8.1749					
100	17.196	2515.5	2687.5	8.4479	3.418	2511.8	2682.5	7.6947	1.0
150	18.512	2587.8	2783.0	8.6882	3.689	2585.6	2780.1	7.9401	1.0
200	21.825	2681.3	2879.5	8.9038	4.356	2659.9	2877.7	8.1580	2.0
250	24.136	2736.0	2977.3	9.1002	4.820	2735.0	2976.0	8.3556	2.0
300	26.445	2812.1	3076.5	9.2813	5.284	2811.3	3075.5	8.5373	2.0
400	31.063	2966.9	3278.6	9.6077	6.209	2968.5	3278.9	8.8642	2.0
500	35.679	3132.3	3488.1	9.8978	7.134	3132.0	3488.7	9.1546	2.0
600	40.295	3302.5	3705.4	10.1608	8.057	3302.2	3705.1	9.4178	2.0
700	44.911	3479.6	3928.7	10.4028	8.981	3479.4	3928.5	9.6599	2.0
800	49.526	3663.8	4159.0	10.6281	9.904	3663.6	4158.9	9.8852	2.0
900	54.141	3855.0	4396.4	10.8396	10.828	3854.9	4396.3	10.0967	2.0
1000	58.757	4053.0	4640.6	11.0393	11.751	4052.9	4640.5	10.2964	2.0
1100	63.372	4257.5	4891.2	11.2287	12.674	4257.4	4891.1	10.4859	2.0
1200	67.987	4467.9	5147.8	11.4081	13.597	4467.8	5147.7	10.6662	2.0
1300	72.602	4683.7	5409.7	11.5811	14.521	4683.6	5409.6	10.8382	2.0

Interpolation is sometimes needed to obtain data not given in the table as shown in the following example.

Example (3-2)

Calculate the specific enthalpy of steam at 98 bar and 360 °C.

Check the quality of steam. From the saturated vapor tables the saturation temperature at 98 bar is 309.5 °C which is less than the actual temperature of steam. Therefore this steam is superheated.

The state of 98 bar and 360 °C is not given in the superheated table. Therefore interpolation is needed. To simplify the interpolation process let us make this table.

P (Bar)	t °C		
	350	360	375
80	2990	A	3069
98		C	
100	2926	B	3019

The required state is (C). Using interpolation find state (A) first, then state (B) and finally find State (B) as shown below.

$$\frac{3069 - h}{3069 - 2990} = \frac{375 - 360}{375 - 350}$$

$\therefore h = 3021.6$ kJ/kg which is the specific enthalpy of state A.

$$\frac{3019 - h}{3019 - 2926} = \frac{375 - 360}{375 - 350}$$

$\therefore h = 2965$ kJ/kg which is the specific enthalpy of state B.

$$\frac{h - 2965}{3021.6 - 2965} = \frac{100 - 98}{100 - 80}$$

$\therefore h = 2970.66$ kJ/kg which is the specific enthalpy of state C, the required state.

THE GAS PHASE

To simplify the study of gas phase we divide the gases into three categories:

1-The perfect gas

2-The ideal gas

3-The actual gas

The perfect gas

It is defined as the gas which follows the equation of state of perfect gas and other perfect gas laws
OR the gas with constant specific heats.

The perfect gas equation of state is written as

$$PV = mRT \quad \text{or per unit mass} \quad (3-22)$$

$$Pv = RT \quad \text{where } v \text{ is the specific gas constant} \quad (3-23)$$

$$PV = nR_oT \quad \text{or per unit mole} \quad (3-24)$$

$$PV = R_oT \quad \text{where } R_o \text{ is the universal gas constant} \quad (3-25)$$

The ideal gas

It is defined as the gas which follows the equation of state of perfect gas and other perfect gas laws but whose specific heats vary linearly with temperature.

The actual gas is the gas which is available in nature such as oxygen, nitrogen, hydrogen, carbon dioxide, ... etc

PERFECT GAS LAWS

A set of statements based on experimental data governs the behavior of perfect gases in addition to the equation of state.

1-Boyles law

It is stated as “The volume of a perfect gas is inversely proportional with its pressure at constant temperature”. The mathematical formulation of this law is shown below.

$$V \propto \left(\frac{1}{P} \right)_T \quad (3-26)$$

$$(PV)_T = C \quad (3-27)$$

Where,

P=pressure (Pa)

V=volume (m³)

T=temperature (K)

C=constant (N.m)

The subscript (T) in the above equations refers to constant temperature.

For a process at constant temperature this is written as,

$$P_1V_1 = P_2V_2 = PV = C \quad (3-28)$$

Fig (3-7) represents Boyles law on the P-V diagram

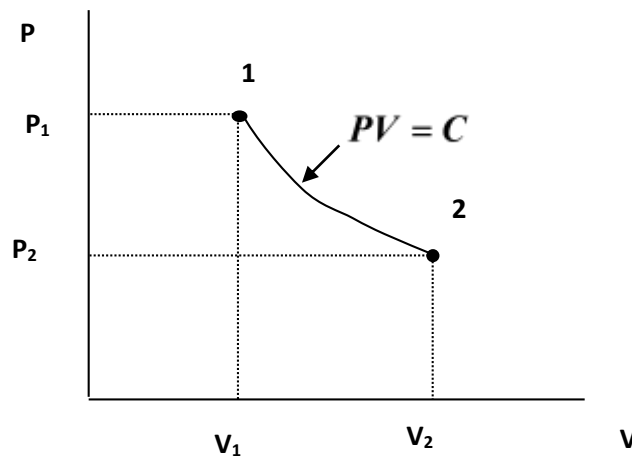


Fig (3-7) Representation of Boyles law

The process represented by Boyles law is known as constant temperature process or **isothermal process**.

2-Charles law

It is stated as follows “The volume of a perfect gas is directly proportional with temperature at constant pressure”. The mathematical formulation of this law is shown below.

$$V \propto (T)_P \quad (3-29)$$

$$\left(\frac{V}{T}\right)_P = C \quad (3-30)$$

$$\frac{V_1}{T_1} = \frac{V_2}{T_2} = \frac{V}{T} = C \quad (3-31)$$

The subscript (P) in the above equations refers to the constant pressure process. This process is also referred to as **isobaric process**. Fig (3-8) shows the process on the P-V diagram.

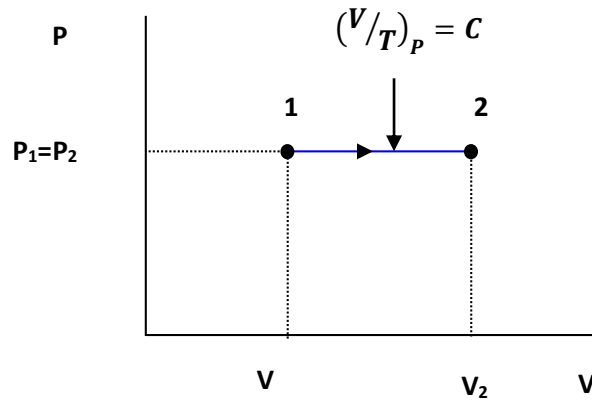


Fig (3-8) Representation of Charles law (isobaric process)

There is another statement for this law “The pressure is directly proportional with temperature at constant volume”. The mathematical formulation of this law is shown below.

$$P \propto (T)_V \quad (3-32)$$

$$\left(\frac{P}{T}\right)_V = C \quad (3-33)$$

$$\frac{P_1}{T_1} = \frac{P_2}{T_2} = \frac{P}{T} = C \quad (3-34)$$

The subscript (V) in the above equations refers to the constant volume process. This process is also referred to as **isochoric process**. Fig (3-9) shows the process on the P-V diagram.

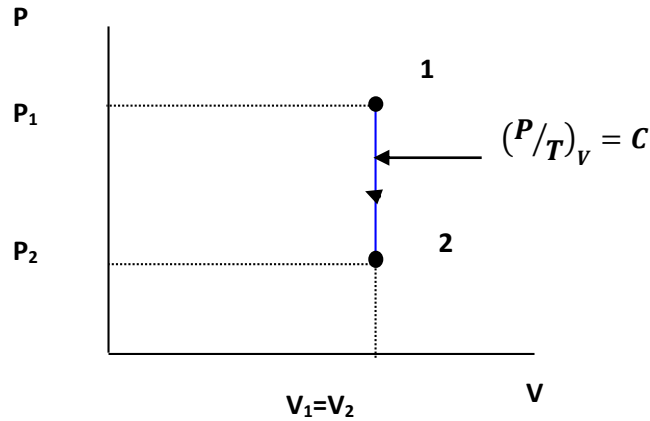


Fig (3-9) Representation of Charles law (isochoric process)

3-EQUATION OF STATE OF PERFECT GAS

This equation can be derived by combining Boyles and Charles laws.

Let us consider a perfect gas compressed isothermally from state (1) to state (2) and then heated at constant pressure from state (2) to state (3) as shown in fig (3-10).

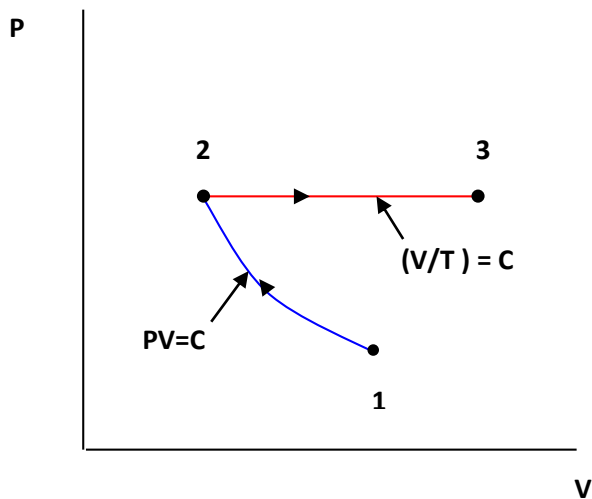


Fig (3-10)

Apply Boyles law for the process (1-2),

$$P_1V_1 = P_2V_2 \quad (3-35)$$

Then apply Charles law for process (2-3),

$$\frac{V_2}{T_2} = \frac{V_3}{T_3} \quad (3-36)$$

From equation (3-35),

$$V_2 = \frac{P_1V_1}{P_2} \quad (3-37)$$

Substitute in equation (3-36)

$$\frac{P_1V_1}{T_2P_2} = \frac{V_3}{T_3} \quad (3-38)$$

By rearranging equation (3-38),

$$\frac{P_1V_1}{T_2} = \frac{P_2V_3}{T_3} \quad (3-39)$$

But we know that,

$$T_1 = T_2, P_2 = P_3, \text{ hence}$$

$$\frac{P_1V_1}{T_1} = \frac{P_3V_3}{T_3} \quad (3-40)$$

Hence we can generalize,

$$\frac{PV}{T} = C \quad (3-41)$$

Where C is constant.

This equation can be written in another form as,

$$PV = CT \quad (3-42)$$

Or per unit mass,

$$PV = mRT \quad (3-43)$$

Where (R) is the specific gas constant and (m) is the mass of gas.

Or per unit mass,

$$Pv = RT \quad (3-44)$$

This equation can also be written in terms of moles as,

$$PV = nR_oT \quad (3-45)$$

Where (n) is the number of moles, or per unit mole,

$$Pv = R_oT \quad (3-46)$$

Where (R_o) is the universal gas constant.

Where (MW) is the molecular weight.

The value of universal gas constant can be calculated from Avogadro's law which is stated as "equal volumes of gases contain same number of molecules at the same pressure and temperature".

$$PV_m = R_oT \quad (3-47)$$

Where V_m is the volume of one mole which is the same for all gases at same pressure and temperature.

$$V_m = 22.4145 \text{ m}^3/\text{mole}$$

Therefore at a pressure of 101325 Pa and a temperature of 273.15 K the value of R_o is,

$$R_o = \frac{PV_m}{T} = \frac{101325 \times 22.4145}{273.15} = 8314.66 \text{ J/mole.K}$$

$$n = \frac{m}{MW} \quad (3-48)$$

$$PV = \frac{m}{MW} R_oT \quad (3-49)$$

$$PV = m \frac{R_o}{MW} T \quad (3-50)$$

By comparing equations (3-43) and (3-50) we find that,

$$R = \frac{R_o}{MW} \quad (3-51)$$

The specific gas constant of air can be calculated from the above equation if we assume that the molecular weight of air is 28.97 kg/mole.

$$R = \frac{8314.66}{28.97} = 287.00932 \text{ J/kg.K} \cong 287 \text{ J/kg.K}$$

Example (3-3)

A vertical piston cylinder arrangement contains a quantity of air at a pressure of 200 kPa and a temperature of 500 °C as shown in fig. The piston cross sectional area is 0.2 m². If the cylinder is cooled, calculate:

- 1-The air temperature at the moment when the piston touches the stops.
- 2-The pressure inside the cylinder when the air temperature reaches 20 °C.

Solution

Assume that the air inside the cylinder behaves as a perfect gas.

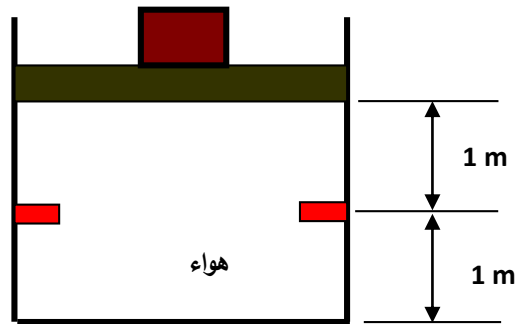
1-The first stage of cooling process is carried out at constant pressure since the piston is always at equilibrium.

$$V_1 = A \times l = 0.2 \times 2 = 0.4 \text{ m}^3$$

$$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$

$$V_2 = A \times l = 0.2 \times 1 = 0.2 \text{ m}^3$$

$$\therefore T_2 = T_1 \frac{V_2}{V_1} = 773 \times \frac{0.2}{0.4} = 386.5 \text{ K}$$



2-The second stage of cooling is at constant volume since the piston rests on the stops.

$$\frac{P_2}{T_2} = \frac{P_3}{T_3}$$

$$P_2 = P_1 = 200 \text{ kPa}$$

$$\therefore P_3 = P_2 \frac{T_3}{T_2} = 200 \times \frac{293}{386.5} = 151.62 \text{ kPa}$$

There are other equations of state to describe the behavior of semi-perfect gases some of them empirical. The perfect gas equation of state cannot describe the gas behavior at high densities. To do so the equation is modified by introducing a factor known as the compressibility factor and the equation becomes:

$$Pv = ZR_oT \quad (3-52)$$

where Z is the compressibility factor.

$$Z = \frac{Pv}{R_oT} \quad (3-53)$$

$$\therefore \rho = \frac{1}{v} \quad (3-54)$$

$$\therefore Z = \frac{P}{\rho R_oT} \quad (3-55)$$

The compressibility factor accounts for the change of density of actual gases. The value of (Z) equals unity for the perfect gas and larger or less than unity for actual gases depending on the gas pressure and temperature.

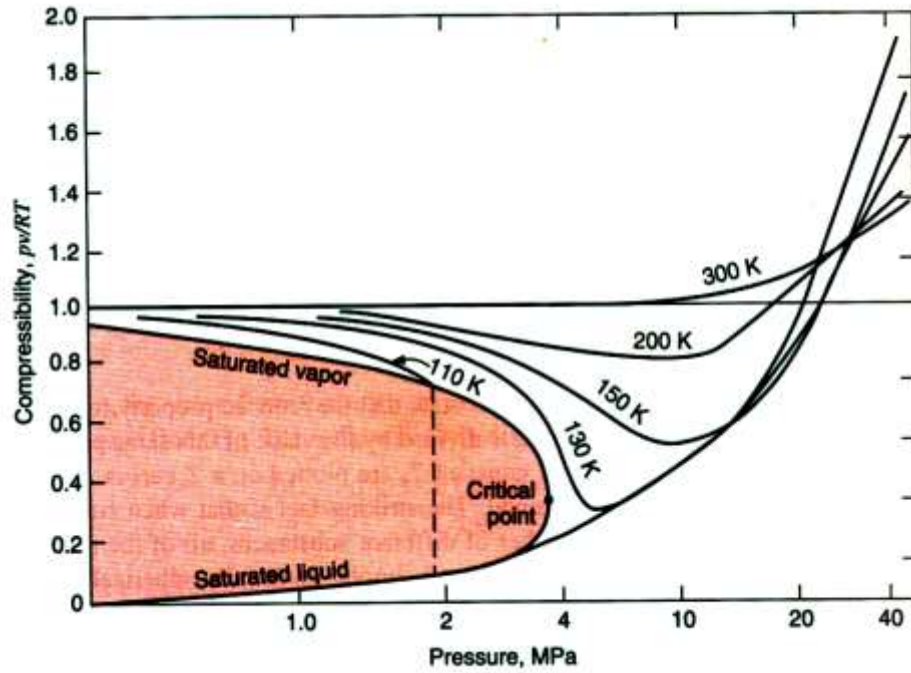


Fig (12) The Compressibility Factor Chart for Nitrogen

There are similar compressibility factor charts for other actual gases. The compressibility factor charts are given in terms of reduced pressure (P_r) and reduced temperature (T_r).

$P_r = \frac{P}{P_c}$ and $T_r = \frac{T}{T_c}$ where P_c and T_c are the critical pressure and critical temperature respectively.

$$Z = \frac{Pv}{R_o T} \quad (3-56)$$

$$P = P_r P_c, \quad T = T_r T_c, \quad v = v_r v_c$$

Substituting in the above equation,

$$Z = \frac{P_r P_c v_r v_c}{R_o T_r T_c}$$

By rearranging the above equation we obtain,

$$Z = \frac{P_c v_c}{R_o T_c} \frac{P_r v_r}{T_r} \quad (3-57)$$

$$Z_c = \frac{P_c v_c}{R_o T_c} \quad (3-58)$$

Where Z_c is the compressibility factor at the critical point.

$$\therefore Z = Z_c \frac{P_r v_r}{T_r} \quad (3-59)$$

The value of Z_c is between (0.23-0.33) for all actual gases.

There are many empirical equations to describe the behavior of actual gases and the most familiar equation is Van der Waals equation which is shown below.

$$P = \frac{R_o T}{v - b} - \frac{a}{v^2} \quad (3-60)$$

The two constants in the above equation (a and b) are obtained experimentally. The difference between this equation and the perfect gas equation of state is the presence of the two terms $\frac{a}{v^2}$ and b . The first term accounts for the effect of intermolecular forces on pressure while the other term is a correction factor for the actual volume of gas molecules which is equal to the molecule volume multiplied by the number of molecules. The Van der Waals equation at the critical point is written as,

$$P_c = \frac{R_o T_c}{v_c - b} - \frac{a}{v_c^2} \quad (3-61)$$

The first and second derivatives of this equation equal zero since it is an inflection point. Hence,

$$v_c = 3b, \quad P_c = \frac{a}{27b^2}, \quad T_c = \frac{8a}{27R_o b}$$

$$a = \frac{27}{64} \frac{R_o^2 T_c^2}{P_c}, \quad b = \frac{R_o T_c}{8P_c}$$

After substituting the values of P_c , T_c and v_c in equation (3-58), hence

$$Z_c = \frac{3}{8}$$

This value is slightly higher than the value of the compressibility factor for actual gases.

Other empirical equations include:

1-Beatti - Bridgeman equation

$$P = \frac{R_o T(1-\varepsilon)}{v^2} (v + B) - \frac{A}{v^2} \quad (3-62)$$

2- Theoretical equation of state

$$Z = \frac{Pv}{R_o T} = 1 + \frac{B(T)}{v} + \frac{C(T)}{v^2} + \frac{D(T)}{v^3} + \dots \quad (3-63)$$