

First Law of Thermodynamics and Flow Processes

Prof Haroun A K Shahad

Flow processes occur in the open system where mass and energy cross its boundary during a period of time. To simplify the analysis of these processes the term **control volume** is introduced. It is defined as a volume in space defined by certain coordinates and specified for particular analysis and enclosed by an imaginary surface called the control surface. Heat and work cross the control surface. The flow processes are of two types:

1-Steady flow processes,

2-Unsteady flow processes.

A steady flow process is a process in which matter and energy flow in and out of an open system at steady rates. Moreover, an open system undergoing a steady flow process does not experience any change in the mass and energy of the system in addition to fluid properties.

The unsteady flow process is the process in which mass and energy within the control volume in addition to fluid properties vary continuously as the fluid flows into and out of the system.

Both steady and unsteady flow processes may reversible or irreversible. When the first law of thermodynamics is applied to flow processes the obtained equation is called the **Flow Energy Equation** OR (energy conservation equation) which is associated with another equation known as the **Continuity Equation** OR (mass conservation equation). The derivation of these two equations is explained in the next sections.

7-1 Continuity Equation and Control Volume

To simplify the derivation of this equation let us consider the open system and the control volume shown in fig (7-1).

Assume a mass δm_i enters the control volume and a mass δm_e leaves the control volume during a period of time δt .

The mass within the control volume at time (t) = m_t

Total mass of system = $m_t + \delta m_i$

After time δt ,

The mass within the control volume at time $t + \delta t$ = $m_{t+\delta t}$

Total mass of system at time $t + \delta t$ = $m_{t+\delta t} + \delta m_e$

Apply the mass conservation equation for this system,

$$m_t + \delta m_i = m_{t+\delta t} + \delta m_e \quad (7-1)$$

$$\delta m_i - \delta m_e = m_{t+\delta t} - m_t \quad (7-2)$$

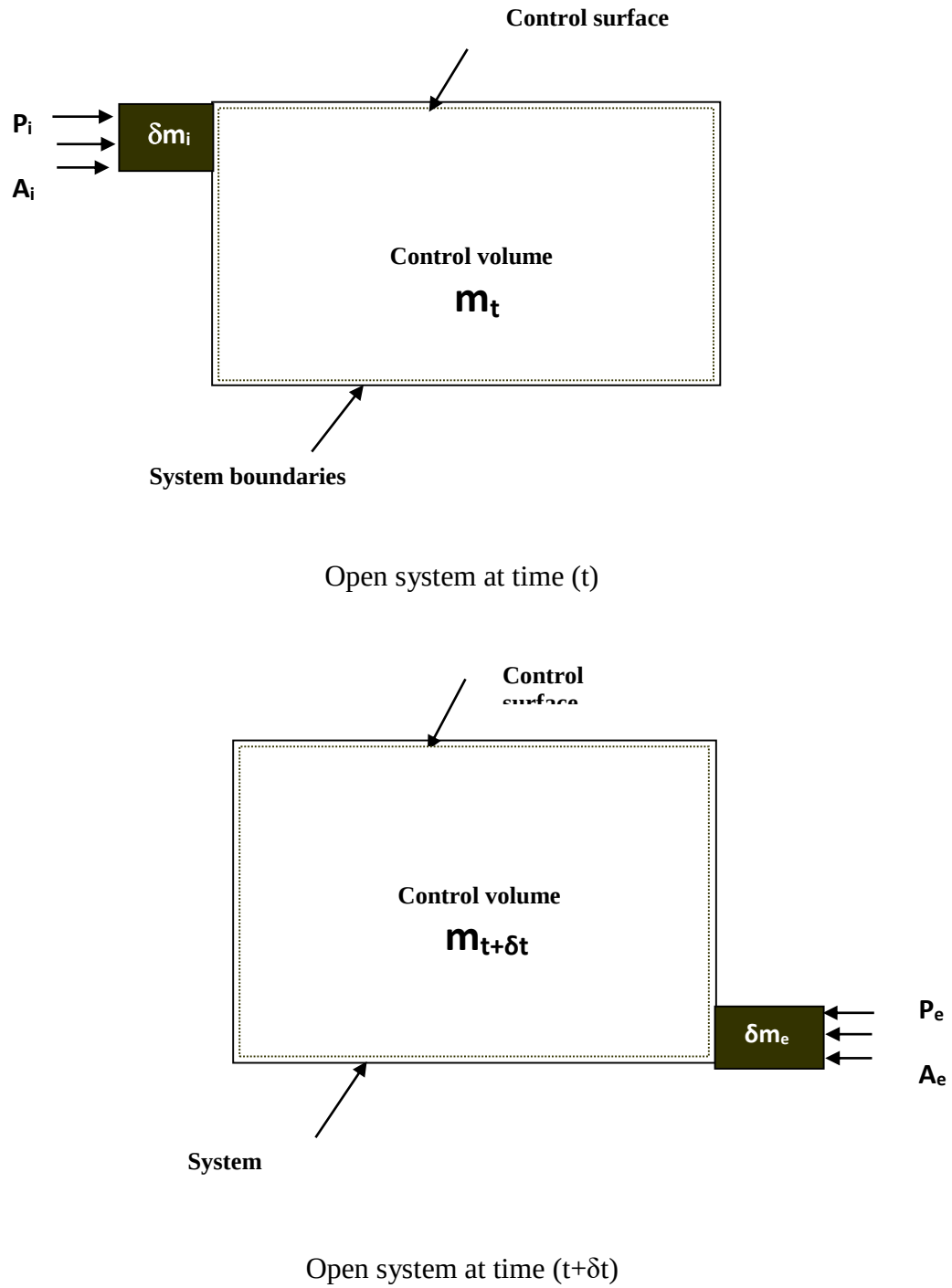


Fig (7-1) Representation of the open system and control volume

It is clear from equation (7-2) that the change of mass within the control volume equals the net flow into and out of the system.

$$(m_{t+\delta t} - m_t) + (\delta m_e - \delta m_i) = 0 \quad (7-3)$$

The time rate is obtained by dividing equation (7-3) by the time duration δt .

$$\frac{(m_{t+\delta t} - m_t)}{\delta t} + \frac{\delta m_e}{\delta t} - \frac{\delta m_i}{\delta t} = 0 \quad (7-4)$$

As δt approaches zero equation (7-4) becomes,

$$\lim_{\delta t \rightarrow 0} \left\{ \frac{m_{t+\delta t} - m_t}{\delta t} \right\} + \lim_{\delta t \rightarrow 0} \left(\frac{\delta m_e}{\delta t} \right) - \lim_{\delta t \rightarrow 0} \left(\frac{\delta m_i}{\delta t} \right) = 0 \quad (7-5)$$

$$\lim_{\delta t \rightarrow 0} \left\{ \frac{m_{t+\delta t} - m_t}{\delta t} \right\} = \frac{dm_{cv}}{dt}, \quad \lim_{\delta t \rightarrow 0} \left(\frac{\delta m_e}{\delta t} \right) = \frac{dm_e}{dt},$$

$$\lim_{\delta t \rightarrow 0} \left(\frac{\delta m_i}{\delta t} \right) = \frac{dm_i}{dt}$$

$$\therefore \frac{dm_{cv}}{dt} + \frac{dm_e}{dt} - \frac{dm_i}{dt} = 0 \quad (7-6)$$

m_{cv} = instantaneous mass within the control volume

$\frac{dm_{cv}}{dt}$ = rate of mass change within the control volume

$\frac{dm_e}{dt}$ = rate of mass flow out of the control volume = $\dot{m}_e$

$\frac{dm_i}{dt}$ = rate of mass flow into the control volume = $\dot{m}_i$

$$\therefore \frac{dm_{cv}}{dt} + \dot{m}_e - \dot{m}_i = 0 \quad (7-7)$$

For more than one inlet and more than one exit for the system equation (7-7) is written as,

$$\frac{dm_{cv}}{dt} + \sum \dot{m}_e - \sum \dot{m}_i = 0 \quad (7-8)$$

This equation is used for both steady and unsteady flow processes. In steady flow processes there is no change of mass within the control volume with time (the mass within the control volume is constant). Therefore, the value of the term $\frac{dm_{cv}}{dt}$ equals zero.

$$\sum \dot{m}_e - \sum \dot{m}_i = 0$$

$$\sum \dot{m}_e = \sum \dot{m}_i \quad (7-9)$$

For one inlet and one exit $\dot{m}_i = \dot{m}_e$

Equation (7-9) is the continuity equation for steady flow processes which indicates that the rate of mass flow into the control volume equals the rate of mass flow out of the control volume.

The mass flow rate is expressed as,

$$\dot{m} = \rho AC \quad (7-10)$$

$\dot{m}$ = mass flow rate (kg/s)

ρ = fluid density (kg/m³)

A = cross sectional area (m²)

C = fluid flow velocity (m/s)

7-2 Law of Energy Conservation and Control Volume

The following formulation of the first law of thermodynamics is valid for all flow and non-flow processes.

$$Q = (E_2 - E_1) + W \quad (7-11)$$

$$E = U + K.E + P.E \quad (7-12)$$

E = fluid intrinsic energy

U = fluid internal energy

KE = fluid kinetic energy

PE = fluid potential energy

Let us consider the control volume shown in fig (6-2) to derive the law of energy conservation starting with equation (7-11).

Assume a mass δm_i enters the control volume during a period of time δt through a cross sectional area A_i and a mass δm_e leaves the control volume during the same period through a cross sectional area A_e . Also assume that an amount of heat δQ and an amount of work δW_{cv}

cross the control surface during the same period as shown in fig (7-2). Equation (7-11) can be written in differential form as below.

$$\delta Q = (E_2 - E_1) + \delta W \quad (7-13)$$

Divide equation (7-13) by δt .

$$\frac{\delta Q}{\delta t} = \frac{(E_2 - E_1)}{\delta t} + \frac{\delta W}{\delta t} \quad (7-14)$$

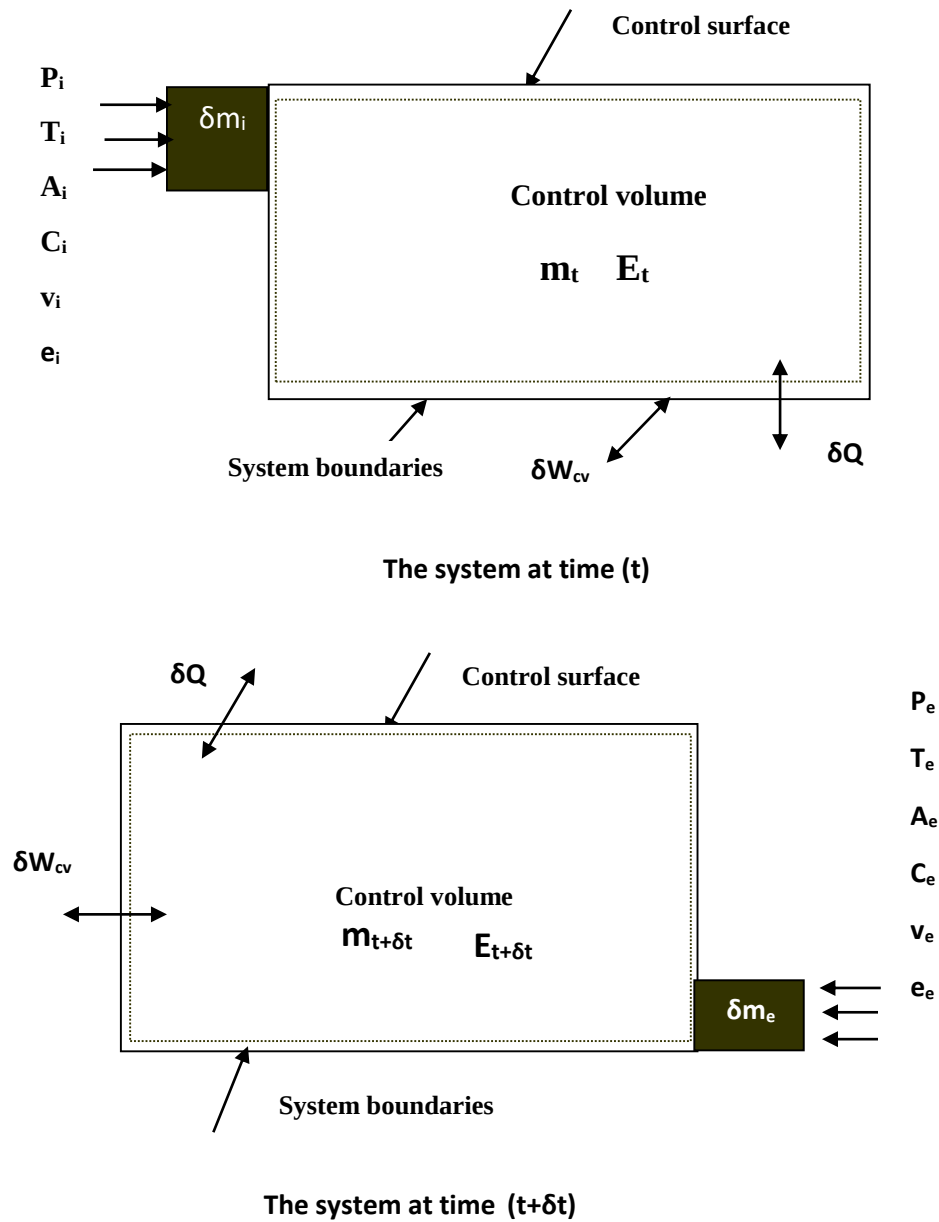


Fig (7-2) Control Volume

$$\frac{\delta Q}{\delta t} = \text{rate of heat transfer to or from the system}$$

$$\frac{(E_2 - E_1)}{\delta t} = \text{rate of change of total energy within the system}$$

$$\frac{\delta W}{\delta t} = \text{rate of work done by or on the system}$$

Assume that,

E_t = total energy within the control volume at time (t)

$E_{t+\delta t}$ = total energy within the control volume at time (t+ δt)

E_1 = total energy of system at time (t)

E_2 = total energy of system at time (t+ δt)

$$E_1 = E_t + e_i \delta m_i$$

$$E_2 = E_{t+\delta t} + e_e \delta m_e$$

$$\therefore E_2 - E_1 = (E_{t+\delta t} - E_t) + (e_e \delta m_e - e_i \delta m_i) \quad (7-15)$$

When the mass δm_i enters the system through the area A_i a quantity of work equals $\delta m_i P_i v_i$ is done by the surroundings on the system and when the mass δm_e leaves the system through the area A_e a quantity of work equals $\delta m_e P_e v_e$ is done by the system on the surroundings. These two quantities of work are known as the flow work. Therefore the total work done is,

$$\delta W = \delta W_{cv} + \delta m_e P_e v_e - \delta m_i P_i v_i \quad (7-16)$$

Substitute equations (7-15 and 7-16) in equation (7-14) and rearrange the new equation,

$$\frac{\delta Q}{\delta t} + \frac{\delta m_i}{\delta t} (e_i + P_i v_i) = \frac{E_{t+\delta t} - E_t}{\delta t} + \frac{\delta m_e}{\delta t} (e_e + P_e v_e) + \frac{\delta W_{cv}}{\delta t} \quad (7-17)$$

$$\text{But, } e = u + \frac{C^2}{2} + Zg$$

$$\frac{\delta Q}{\delta t} + \frac{\delta m_i}{\delta t} \left(u_i + \frac{C_i^2}{2} + Z_i g + P_i v_i \right) = \frac{E_{t+\delta t} - E_t}{\delta t} + \frac{\delta m_e}{\delta t} \left(u_e + \frac{C_e^2}{2} + Z_e g + P_e v_e \right) + \frac{\delta W_{cv}}{\delta t} \quad (7-18)$$

The term $(u+Pv)$ equals the enthalpy of the fluid (h) .

As the value of δt approaches zero equation (7-18) becomes,

$$\dot{Q} + \dot{m}_i \left(h_i + \frac{C_i^2}{2} + Z_i g \right) = \frac{dE_{cv}}{dt} + \dot{m}_e \left(h_e + \frac{C_e^2}{2} + Z_e g \right) + \dot{W}_{cv} \quad (7-19)$$

Equation (7-19) is the energy conservation equation for the control volume (the first law of thermodynamics). This equation is a general energy equation for both closed and open system since when $\dot{m}_i = \dot{m}_e = 0$ there is no flow into or out of the system and it becomes a closed system.

$$\dot{Q} = \frac{dE_{cv}}{dt} + \dot{W}_{cv} \quad (7-20)$$

Equation (7-20) is similar to the energy equation for closed system derives in chapter six.

The term $\frac{dE_{cv}}{dt}$ represents the rate of change of total energy within the closed system.

Equation (7-19) is used for all reversible and irreversible flow processes.

For steady flow processes the mass flow rate into the system equals the mass flow rate out of the system which means that the mass within the control volume remains constant.

$$\dot{m}_i = \dot{m}_e$$

For more than one inlet and more than one exit,

$$\sum \dot{m}_i = \sum \dot{m}_e$$

$$\frac{dm_{cv}}{dt} = 0$$

Some of the steady flow processes are classified as steady state steady flow processes (SSSF) when the following conditions are satisfied:

1-The control volume is stationary.

2-The state of the fluid within the control volume is independent of time.

3-The mass flux (mass flow rate per unit area) on the control surface is independent of time.

4-The rates of heat transfer and work done across the control surface are constant.

5-The following relationships are valid.

$$\frac{dm_{cv}}{dt} = 0, \quad \frac{dE_{cv}}{dt} = 0, \quad \sum \dot{m}_i = \sum \dot{m}_e = \sum \dot{m}$$

Therefore equation (7-19), for one inlet and one exit is written as,

$$\dot{Q} + \dot{m} \left(h_i + \frac{C_i^2}{2} + Z_i g \right) = \dot{m} \left(h_e + \frac{C_e^2}{2} + Z_e g \right) + \dot{W}_{cv} \quad (7-21)$$

This is the energy equation for all reversible and irreversible steady flow processes.

$$q + h_i + \frac{C_i^2}{2} + Z_i g = h_e + \frac{C_e^2}{2} + Z_e g + W \quad (7-22)$$

Equation (7-22) is the steady flow energy equation per unit mass of fluid.

Example 7-1

A radial flow compressor is used to compress air from 1.05 bar and 15 °C to 2 bar and 97 °C. The mass flow rate of air is 50 kg/min. Calculate the power required to drive the compressor and the power required to drive the compressor if the process is frictionless.

This is an irreversible steady flow process and equation (7-21) can be used.

$$\dot{Q} + \dot{m} \left(h_i + \frac{C_i^2}{2} + Z_i g \right) = \dot{m} \left(h_e + \frac{C_e^2}{2} + Z_e g \right) + \dot{W}_{cv}$$

Assume that the process is adiabatic ($\dot{Q} = 0$)

Also assume that the changes in KE and PE are negligible.

Hence the above equation becomes,

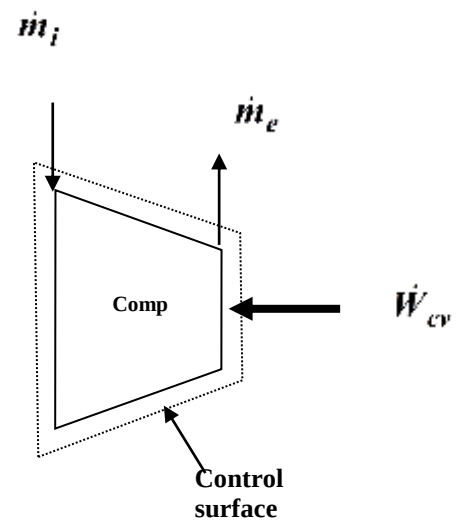
$$\dot{m}_i(h_i) = \dot{m}_e(h_e) + \dot{W}_{cv}$$

$$\dot{m}_i = \dot{m}_e = \dot{m}$$

$$\therefore \dot{W}_{cv} = \dot{m}(h_i - h_e) = \dot{m}c_p(T_i - T_e)$$

The term $\dot{W}_{cv}$ is the power required to drive the compressor.

$$\therefore \dot{W}_{cv} = \frac{50}{60} \times 1.005 \times (288 - 370) = -68.675 \text{ kW}$$



A frictionless process means reversible process. In this case the exit temperature is calculated from the reversible adiabatic relations (isentropic) for perfect gas.

$$T_2 = T_1 \left(\frac{P_2}{P_1} \right)^{\frac{\gamma-1}{\gamma}}$$

$$\therefore T_2 = 288 \times \left(\frac{2}{1.05} \right)^{\frac{0.4}{1.4}} = 346.28 \text{ K}$$

Hence the power required to drive the compressor for the frictionless process will be,

$$\dot{W}_{cv} = \frac{50}{60} \times 1.005 \times (288 - 346.28) = -48.8 \text{ kW}$$

7-3 Application of Steady Flow Energy System

There are many practical engineering applications for the steady flow energy equation using different working fluids such as liquids, vapors or gases. Some of the processes that occur in these applications may be assumed reversible with an acceptable error percentage. When the process is irreversible the practical efficiency should be accounted for. Some of these applications are analyzed in the following sections.

Boiler and Condenser

From the thermodynamic point of view the condenser is the inverse of the boiler. Both of them are heat transfer devices. The working fluid is heated in the boiler while it is cooled in the condenser. In the boiler the working fluid is initially liquid and converted to vapor by heating while in the condenser the working fluid is initially vapor and converted to liquid by cooling. Schematic representations are shown in figs (7-3 & 7-4) for boiler and condenser respectively.

Since both boiler and condenser are heat transfer devices no work is done during heating or cooling processes ($W=0$). The steady flow energy equation per unit mass becomes,

$$q + h_i + \frac{C_i^2}{2} + Z_i g = h_e + \frac{C_e^2}{2} + Z_e g \quad (7-23)$$

The change in potential energy (PE) of the working fluid as it flows through the boiler and condenser is very small and always neglected while the change in kinetic energy (KE) of the working fluid is also small but sometimes is accounted for. If the changes in PE and KE are neglected equation (7-23) becomes,

$$q = h_e - h_i \quad (7-24)$$

For a mass flow $\dot{m}$ equation (7-24) is written as,

$$\dot{Q} = \dot{m}(h_e - h_i) \quad (7-25)$$

Where $\dot{Q}$ equals the rate of heat exchange.

Since the velocity of the fluid inside the boiler or the condenser is small the friction losses are also small and the process may be assumed internally reversible.

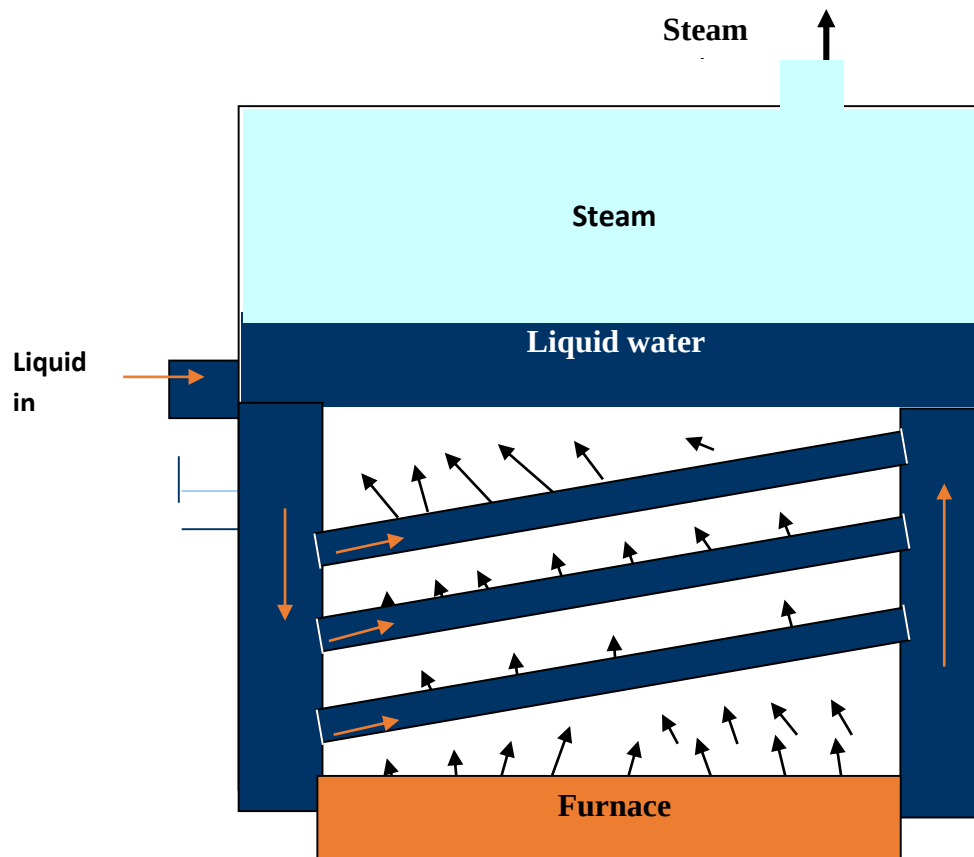


Fig (7-3) Schematic Representation of Boiler

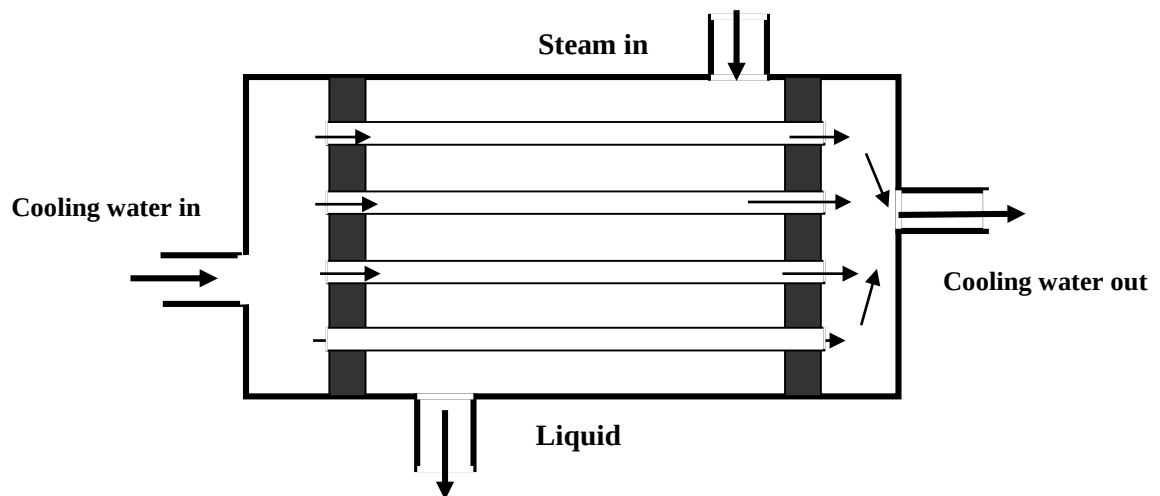
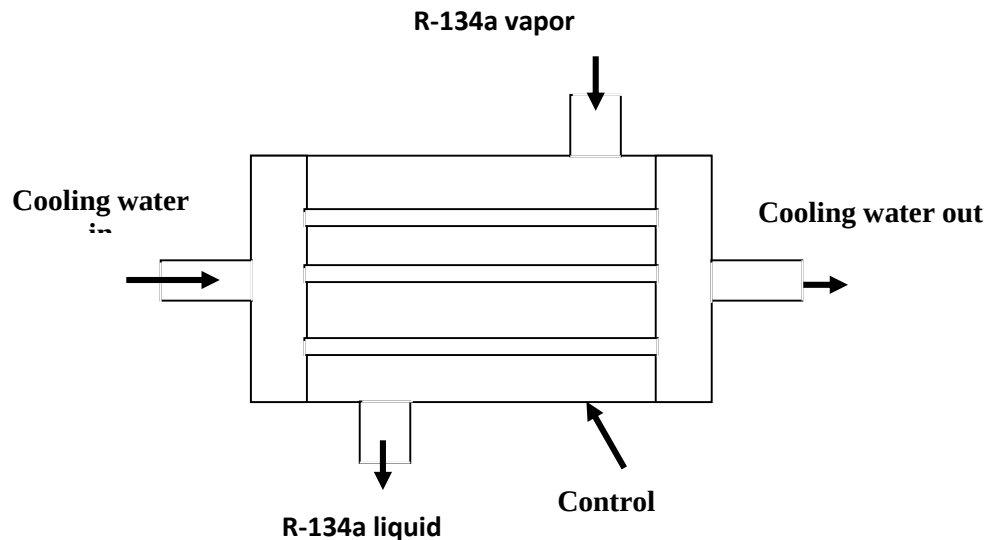


Fig (7-4) Schematic Representation of Condenser

Example 7-2

A condenser in a large refrigeration unit uses R-134a as a refrigerant. The condenser is cooled by liquid water. The refrigerant enters the condenser at a pressure of 1 MPa and 60 °C and leaves at a pressure of 0.95 MPa and a temperature of 35 °C. The refrigerant flow rate is 0.2 kg/s. The cooling water enters at 10 °C and leaves at 20 °C. Calculate the cooling water flow rate.

Consider the control volume shown in fig.



Let us make the following assumptions to simplify the solution.

1-Changes in KE and PE of refrigerant are neglected.

2-The heat lost by refrigerant is gained by the cooling water and no heat is exchanged with the surroundings ($Q=0$).

3-The process is SSSF.

As mentioned earlier no work is done by the working fluid in the condenser ($W=0$).

The steady flow energy equation becomes,

$$\sum \dot{m}_i h_i = \sum \dot{m}_e h_e$$

$$\dot{m}_w h_{wi} + \dot{m}_R h_{Ri} = \dot{m}_w h_{we} + \dot{m}_R h_{Re}$$

The subscript (w) refers to cooling water while the subscript R refers to the refrigerant. From the vapor table of the refrigerant find the enthalpy of refrigerant at inlet and exit.

$$h_{Ri} = 441.89 \text{ kJ/kg} \quad , \quad h_{Re} = 249.1 \text{ kJ/kg}$$

From vapor tables of water find the enthalpy of water at inlet and exit.

$$h_{wi} = 42 \text{ kJ/kg} \quad , \quad h_{we} = 83.95 \text{ kJ/kg}$$

Substitute in the above equation to obtain the mass flow of cooling water.

$$\therefore \dot{m}_w = \frac{\dot{m}_R (h_{Ri} - h_{Re})}{(h_{we} - h_{wi})} = \frac{0.2 \times (441.89 - 249.1)}{(83.95 - 42)} \\ = 0.919 \text{ kg/s}$$

This example can be solved by calculating the heat lost by the refrigerant and the heat gained by the cooling water and equating both heats to obtain the mass flow rate of cooling water.

Turbine and Rotary Compressor

Both turbine and compressor are open and steady flow systems. The working fluid is expanded in the turbine to produce work while it is compressed in the compressor by doing work from the surroundings. Hence the thermodynamic analysis of turbine and compressor is similar except the sign of work done, it is positive in the turbine and negative in the compressor. The working fluid is accelerated in the turbine by nozzles and blades while it is decelerated in the compressor by the diffusers.

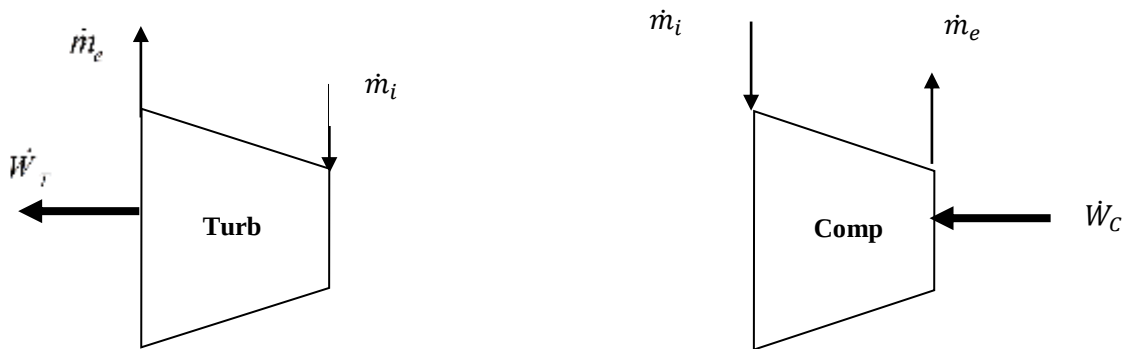


Fig (7-5) Schematic Representation of Turbine and Compressor

The following assumptions are made when applying the steady flow energy equation for turbine and compressor.

- 1-The flow is assumed adiabatic since the flow velocity is high ($Q=0$).
- 2-The fluid velocity at exit is approximately equals that at inlet and hence the change in KE neglected, ($C_1 \cong C_2$).
- 3-The change in PE is also neglected.

Accounting for these assumptions the steady flow energy equation for turbine and compressor becomes.

$$w = h_1 - h_2 \quad (7-26)$$

If the mass flow rate of the fluid is ($\dot{m}$), then the time rate of work done (power) equals,

$$\dot{W} = \dot{m}(h_1 - h_2) \quad (7-27)$$

Where,

$\dot{W}$ =rate of work done or power (kJ/s)

$\dot{m}$ =fluid mass flow rate (kg/s)

h_1 = specific fluid enthalpy at inlet (kJ/kg)

h_2 =specific fluid enthalpy at exit (kJ/kg)

If the working fluid is vapor h_1 and h_2 are found from vapor tables according to quality of vapor while if the working fluid is perfect gas then the enthalpy is calculated using the equation, $h = c_p T$

Therefore, turbine and compressor with perfect gas as the working fluid the energy equation becomes,

$$\dot{W} = \dot{m}c_p(T_1 - T_2) \quad (7-28)$$

$$\dot{W} = \dot{m}c_p T_1 \left(1 - \frac{T_2}{T_1} \right)$$

When the expansion or compression process of the perfect gas is assumed as reversible and adiabatic (isentropic) then the adiabatic ratios equations can be used.

$$\therefore \frac{T_2}{T_1} = \left(\frac{P_2}{P_1} \right)^{\frac{\gamma-1}{\gamma}}$$

$$\therefore \dot{W} = \dot{m}c_p T_1 \left[1 - \left(\frac{P_2}{P_1} \right)^{\frac{\gamma-1}{\gamma}} \right] \quad (7-29)$$

The ratio $\frac{P_2}{P_1}$ represents the expansion or compression ratio in turbine or compressor respectively.

Example 7-3

A small air turbine produces a power of 0.1 kW. Air enters the turbine at 400 kPa and 25 °C and leaves at 100 kPa and -50 °C . Assume that the change in KE of air through the turbine is negligible calculate the required mass flow rate of air to produce such power.

Assume that the process is SSSF and the steady flow energy equation is applied.

$$\therefore C_1 \cong C_2$$

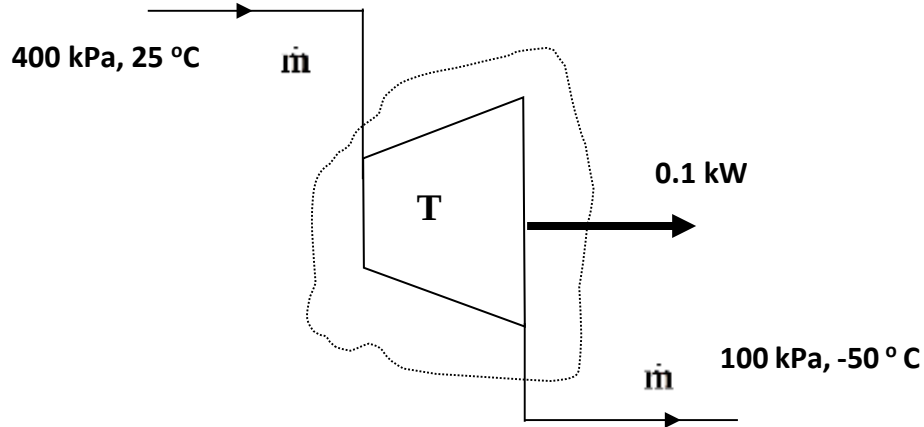
$$\dot{m}h_1 = \dot{m}h_2 + \dot{W}$$

$$\therefore \dot{W} = \dot{m}(h_1 - h_2) = \dot{m}c_p(T_1 - T_2)$$

$$\dot{m} = \frac{\dot{W}}{c_p(T_1 - T_2)}$$

$$\dot{m} = \frac{0.1}{1.005 \times (298 - 223)} = 0.00133 \text{ kg/s}$$

This is the mass flow rate for the actual process.



If the process is assumed as isentropic we can calculate the exit velocity of air as follows.

$$T_2 = T_1 \left(\frac{P_2}{P_1} \right)^{\frac{\gamma-1}{\gamma}} = 298 \times \left(\frac{100}{400} \right)^{\frac{0.4}{1.4}} = 200.5 \text{ K}$$

$$= -72.5 \text{ } ^\circ\text{C}$$

The air mass flow rate for the isentropic process is,

$$\dot{m} = \frac{0.1}{1.005 \times (298 - 200.5)} = 0.001 \text{ kg/s}$$

This is less than that for the real process.

Nozzle and Diffuser

The nozzle is a variable cross sectional area duct used accelerate the working fluid which causes a pressure reduction. The diffuser is the same as the nozzle but the working fluid enters from the larger cross sectional area end. It is used to decelerate the working fluid and raise its pressure. The two devices are shown in fig (7-6).

The nozzle is known also known convergent nozzle and the diffuser as divergent nozzle. When the convergent and divergent nozzles are connected from the narrowest ends the new configuration is known as convergent-divergent nozzle. The connection region has the minimum cross sectional area and is known as the throat.

To simplify the thermodynamic analysis the following assumptions are made.

- 1-No work is done as the working fluid flows through the nozzle and diffuser ($W=0$).
- 2-The flow process is adiabatic since the working fluid velocity is high and no time is available for heat transfer ($Q=0$).
- 3-The change in PE of the working fluid as it flows through the nozzle and diffuser is neglected.

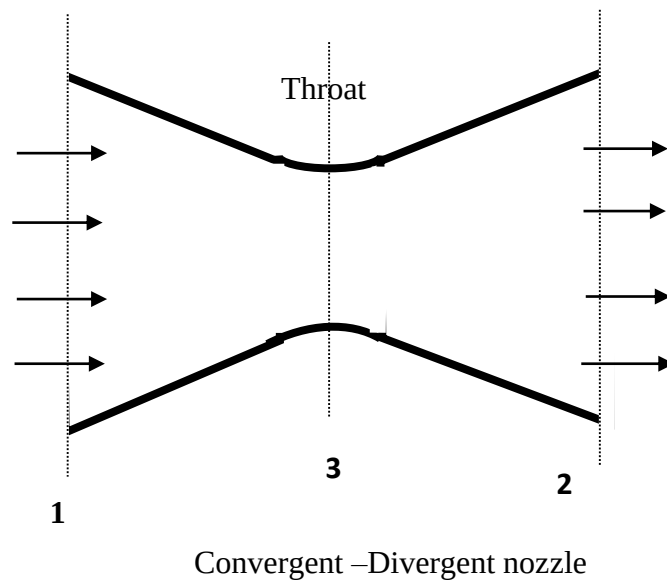
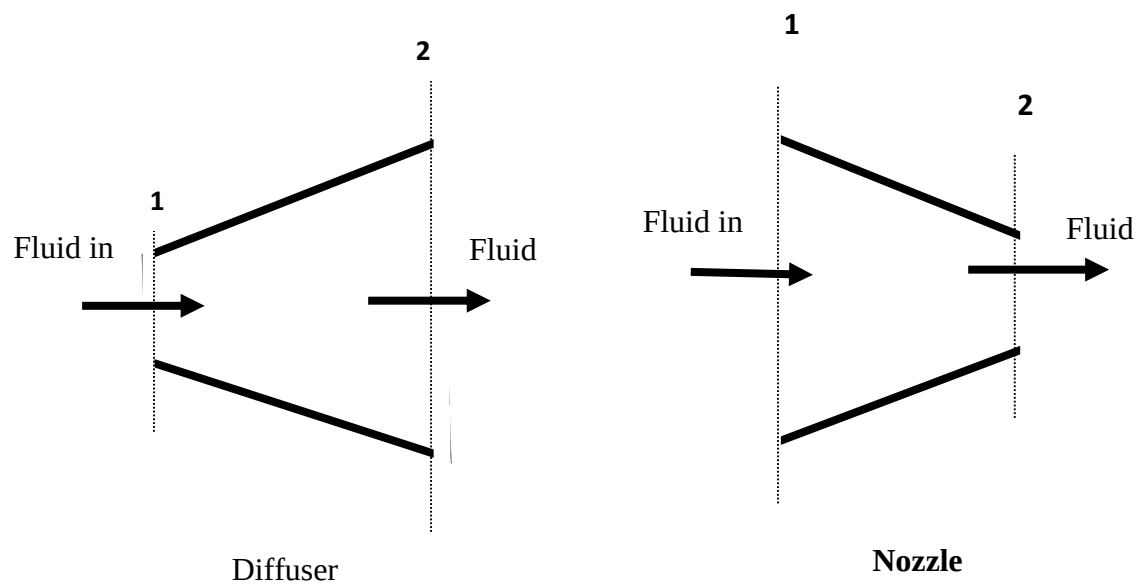


Fig (7-6) Schematic Representation of Nozzle and Diffuser

Accounting for these assumptions the steady flow energy equation becomes,

$$h_1 + \frac{C_1^2}{2} = h_2 + \frac{C_2^2}{2} \quad (7-30)$$

$$\therefore (h_1 - h_2) = \frac{1}{2} (C_2^2 - C_1^2) \quad (7-31)$$

Equation (7-31) indicates that the change in enthalpy of the working fluid equals the change in its kinetic energy (KE).

If the working fluid is vapor then the change in its enthalpy is obtained from the vapor tables according to its quality and properties while if its perfect gas then the isentropic relations for perfect gas are used in the solution and other gas laws.

The continuity equation is also used in the solution.

$$\rho_1 A_1 C_1 = \rho_2 A_2 C_2 \quad (7-32)$$

$$\frac{A_1 C_1}{v_1} = \frac{A_2 C_2}{v_2} \quad (7-33)$$

Example 7-4

A wet steam at 11 bar with a dryness fraction of 0.95 flows through a nozzle. The steam leaves the nozzle at 1 bar. The flow process follows the law $Pv^n = C$. Calculate the value of the index (n), the steam exit velocity and the mass flow rate per unit area assuming that the flow process is isentropic.

This process is SSSF process and the steady flow energy equation is used in the solution.

Since no work is done and no heat transfer during the process the steady flow energy equation becomes,

$$h_1 + \frac{1}{2} C_1^2 = h_2 + \frac{1}{2} C_2^2$$

Obtain the properties of wet steam at nozzle inlet from table,

$$h_1 = h_f + x h_{fg}$$

$$\therefore h_1 = 781.34 + 0.95 \times 2000.4 = 2681.72 \text{ kJ/kg}$$

$$v_1 = x v_g = 0.95 \times 0.1775 = 0.1687 \text{ m}^3/\text{kg}$$

$$s_1 = s_f + x s_{fg} = 2.1792 + 0.95 \times 4.3744 = 6.335 \text{ kJ/kg.K}$$

Since the process is isentropic, $s_2 = s_1$,

$$\therefore s_2 = 6.335 \text{ kJ/kg.K}$$

From saturated steam table find s_g at 1 bar, $s_g = 7.3594 \text{ kJ/kg.K}$

Since $s_2 < s_g$ the steam remains wet at nozzle exit.

$$x = \frac{s_2 - s_f}{s_{fg}} = \frac{6.335 - 1.3025}{6.0568} = 0.83$$

$$\therefore h_2 = h_f + xh_{fg} = 417.46 + 0.83 \times 2258 = 2291.6 \text{ kJ/kg}$$

$$v_2 = xv_g = 0.83 \times 1.694 = 1.406 \text{ m}^3/\text{kg}$$

$$\therefore P_1 v_1^n = P_2 v_2^n$$

$$\therefore \frac{P_1}{P_2} = \left(\frac{v_2}{v_1} \right)^n$$

$$\ln \left(\frac{P_1}{P_2} \right) = n \ln \left(\frac{v_2}{v_1} \right)$$

$$\ln \left(\frac{11}{1} \right) = n \ln \left(\frac{1.406}{0.1687} \right)$$

$$\therefore n = 1.13$$

If the inlet steam velocity is neglected since it small then the exit velocity can be calculated from the energy equation.

$$C_2 = \sqrt{2(h_1 - h_2)}$$

$$\therefore C_2 = \sqrt{2 \times (2681.72 - 2291.6) \times 1000} = 883.3 \text{ m/s}$$

$$\dot{m} = \frac{A_2 C_2}{v_2}$$

$$\frac{\dot{m}}{A_2} = \frac{C_2}{v_2} = \frac{883.3}{1.406} = 628.25 \text{ kg/s.m}^2$$

Reciprocating Compressor

The reciprocating compressor is a piston cylinder mechanism where the piston reciprocates between two fixed points in the cylinder. The cylinder is fitted with two valves, one for fluid inlet and the other for compressed fluid exit. When the piston moves down the cylinder the pressure drops, the inlet valve is opened and the fluid enters the cylinder. When the piston moves up the cylinder the inlet valve is closed and the pressure rises. When reached the required pressure the exit valve is opened and the compressed fluid leaves the cylinder. Fig (7-7) shows a schematic representation of the reciprocating compressor. The advantages of the reciprocating compressor over the rotary compressor is that it generates high pressures, but it has many disadvantages such as it supplies smaller mass flow rates intermittently due to the presence of valves. Therefore the flow process in the reciprocating compressor is not a steady flow process but it can be assumed

as steady when a reservoir with suitable capacity is used and the steady flow energy equation is valid and can be used in thermodynamic analysis.

In this type of compressor the compression process cannot be assumed as adiabatic since the flow process is slow and the presence of large heat transfer area. The change in PE of the fluid through the compressor is neglected and in some cases the change in KE is also neglected. Hence the steady flow energy equation becomes.

$$h_1 + \frac{1}{2}C_1^2 + q = h_2 + \frac{1}{2}C_2^2 + w \quad (7-34)$$

$$\therefore q - w = h_2 - h_1 \quad (7-35)$$

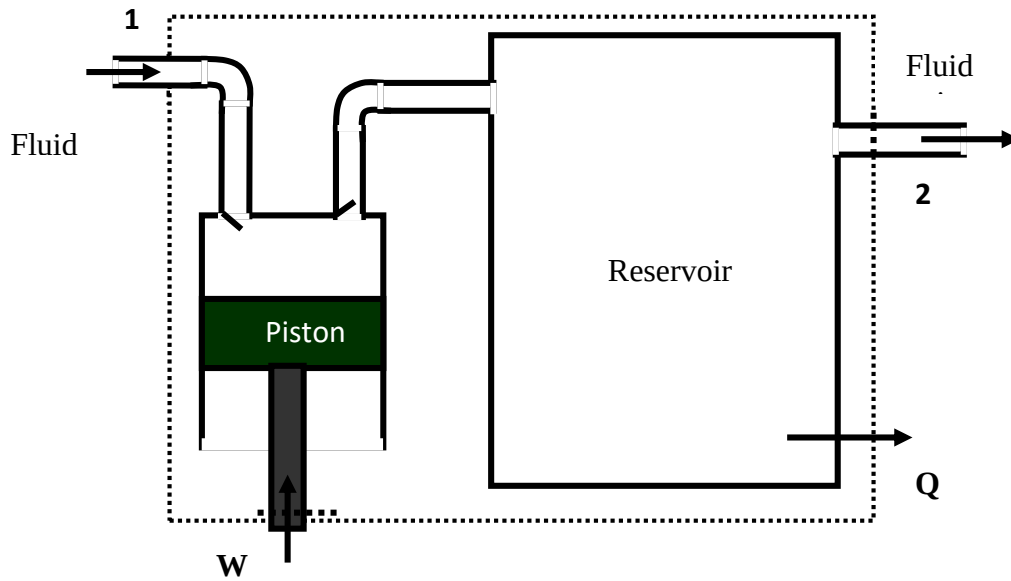


Fig (7-7) Schematic Representation of the Reciprocating Compressor

Throttling Process

The throttling process is a pressure reduction process intentionally by installing a valve in the flow or accidentally due to sedimentation or rust. In both cases the area of flow is reduced as shown in fig (7-8). The throttling process is an irreversible steady flow process. To simplify the thermodynamic analysis the following assumptions are made.

- 1-The changes in KE and PE are neglected.
- 2-No work is done during the process ($W=0$).

3-The throttling process occurs through a short distance in the pipe hence the heat transfer area is small. Therefore the process is assumed adiabatic ($Q=0$)

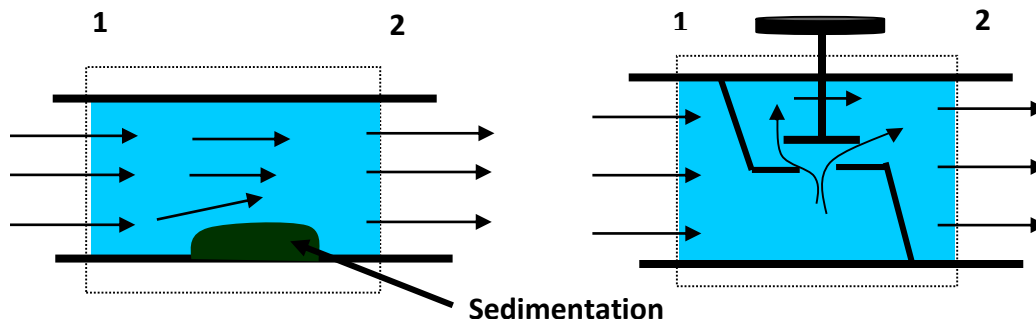


Fig (7-8)

When accounting for these assumptions the steady flow energy equation becomes.

$$h_1 = h_2 \quad (7-36)$$

From this equation (7-36) it is concluded that the throttling process is **isenthalpic process**.

For vapors the enthalpy is obtained from vapor tables according to vapor quality and properties.

For perfect gas the enthalpy is calculated from the following equation.

$$h = c_p T$$

$$\therefore c_p T_1 = c_p T_2$$

$$\therefore T_1 = T_2$$

This means that for perfect gas the throttling process is **isenthalpic and isothermal**. However for actual gas the temperature will change during the throttling process as in the flow through a porous medium, Joule-Thomson experiment and Throttling calorimeter.

Joule-Thomson experiment

This experiment was performed by the scientists Joule and Thomson. In the experiment a fluid flow steadily in a thermally insulated pipe through a porous medium fixed inside the pipe from a high pressure to a low pressure as shown in fig (7-9). The velocity of fluid through the porous medium must be small so that the change in fluid kinetic energy as it flows through the medium is small and can be neglected. The fluid pressure and temperature upstream (P_1 and T_1) and downstream (P_2 and T_2) of the medium are measured to ensure that the fluid is in thermodynamic equilibrium. The pressure P_2 is reduced gradually and the temperature T_2 is measured at each pressure. As indicated previously the fluid enthalpy is constant during the throttling process, therefore the fluid enthalpies upstream and downstream of the porous medium are equal. From these measurements the pressure temperature diagram is plotted and the resulting curve is known

as the isenthalpic line. Many isenthalpic lines are obtained by varying the upstream pressure and temperature as shown in fig (7-10).

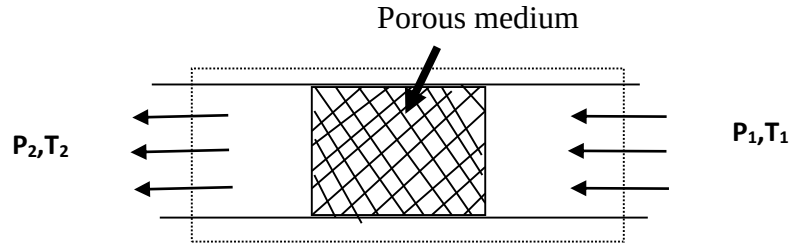


Fig (7-9) Representation of Flow through Porous Medium

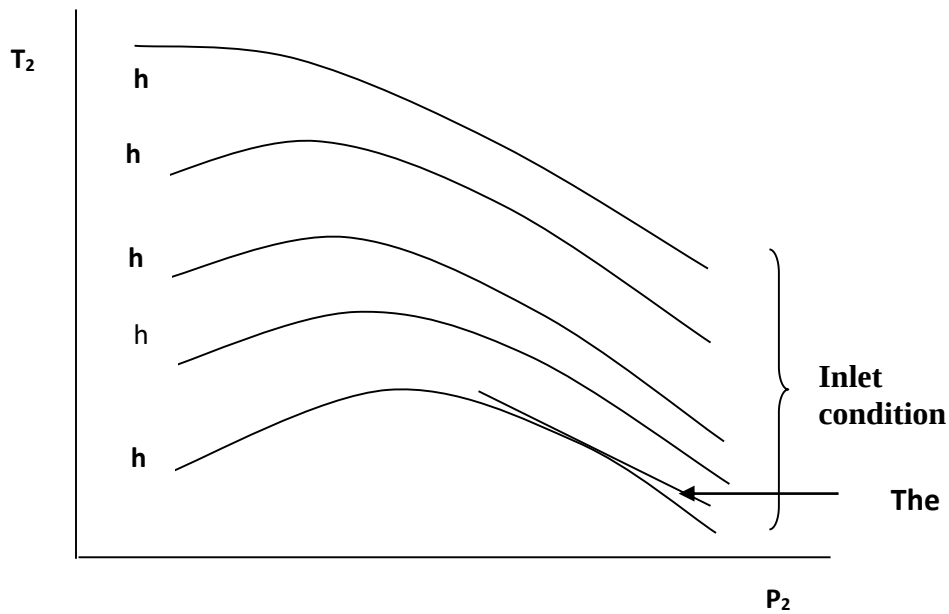


Fig (7-10) Pressure Temperature Diagram

It is observed from fig (7-10) that the slope of the isenthalpic line is either negative, positive or zero. The slope of the isenthalpic line is referred to as Joule Thomson Coefficient and given the symbol (μ). The point of zero slope is known as the inversion point and the pressure and temperature at this point are known as inversion pressure and inversion temperature respectively.

$$\mu = \left(\frac{\partial T}{\partial P} \right)_h \quad (7-37)$$

The curve that joins all inversion points is referred to as the inversion curve.

The negative slope of the isenthalpic line indicates that the fluid temperature increases as it passes through the porous medium as the downstream pressure drops while the positive slope indicates

the fluid temperature drops as it passes through the porous medium as the downstream pressure drops which means the fluid is cooled. This experiment is used in gas liquefaction processes.

Throttling calorimeter

The throttling calorimeter is a device used to determine experimentally the dryness fraction of a vapor. It is mentioned previously that the state of vapor is determined by the knowledge of its pressure and temperature but this is not true for wet vapor since both pressure and temperature of wet vapor are dependent properties. Since the throttling process is an isenthalpic process, according to equation (7-36), the use of the throttling calorimeter determines another property of the wet vapor which is the enthalpy. This calorimeter is shown schematically in fig (7-11).

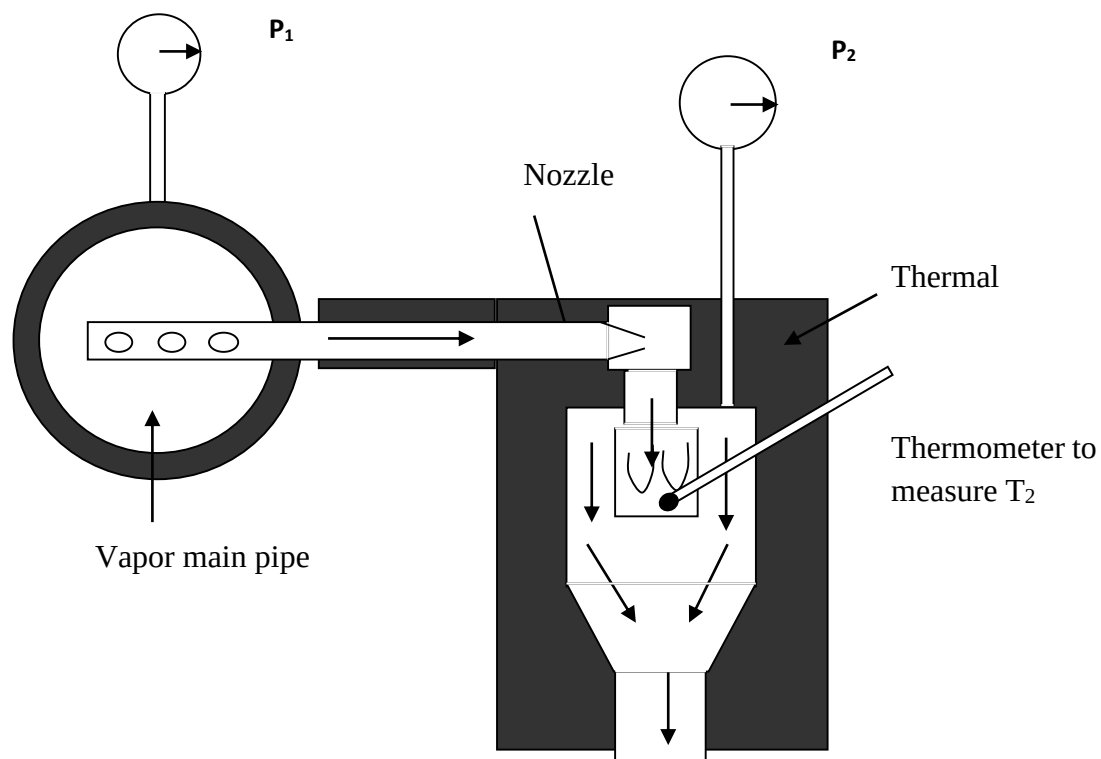


Fig (7-11) Schematic Representation of Throttling Calorimeter

The vapor enters the calorimeter at a pressure P_1 and a temperature T_1 and passes through the calorimeter where it is throttled to a pressure P_2 . The temperature T_2 after throttling is measured. The quality of vapor after throttling must be superheated. The enthalpy of this superheated vapor is obtained from tables knowing its pressure and temperature or from (h-s) diagram. The following relations are used to obtain the dryness fraction.

$$\therefore h_1 = h_2$$

$$h_1 = h_{f1} + xh_{fg1}$$

$$\therefore x = \frac{h_1 - h_{f1}}{h_{fg1}}$$

Where (x) is the dryness fraction of vapor before throttling.

The calorimeter must be thermally insulated ($Q=0$). Also the vapor should not be too wet so that it cannot be superheated after throttling.

Example 7-5

Steam enters a turbine at 40 bar and 300 °C under normal load conditions but due to load reduction the steam must be throttled to 7 bar at turbine inlet. Determine the percentage drop in turbine power due to throttling if the turbine exit pressure is the same in both cases and equals 0.34 bar and the steam flow rate and turbine efficiency are independent of load.

Assume that the expansion process in the turbine in both cases is isentropic.

Determine the quality of steam at turbine inlet and find its enthalpy.

The steam is superheated at turbine inlet ($P=40$ bar and $t=300$ °C). From superheated steam table we find,

$$h_1 = 2960.7 \text{ kJ/kg}$$

$$s_1 = 6.3615 \text{ kJ/kg.K}$$

1-Full load

$$s_2 = s_1 = 6.3615 \text{ kJ/kg.K}$$

Since $s_2 < s_g$ at 0.34 bar the steam is wet at turbine inlet.

$$x = \frac{s_2 - s_f}{s_g - s_f} = \frac{6.3615 - 0.98}{7.73 - 0.98} = 0.798$$

$$h_2 = h_f + xh_{fg} = 3013 + 0.798 \times 2329 = 2158.1 \text{ kJ/kg}$$

$$\therefore \dot{W}_1 = \dot{m}(h_1 - h_2) = \dot{m} \times (2960.7 - 2158.1) = 802.58 \dot{m} \text{ kW}$$

2-Partial load

The throttling process is isenthalpic.

$$h_2 = h_1 = 2960.7 \text{ kJ/kg}$$

Since $h_2 > h_g$ at 7 bar the steam is still superheated.

Knowing the enthalpy and pressure of steam at turbine inlet ($P=7$ bar and $h=2960.7$ kJ/kg) find the entropy and temperature of steam.

$$s_2 = 7.12 \text{ kJ/kg.K}$$

$$\therefore t_2 = 252 \text{ °C}$$

$$s_3 = s_2 = 7.12 \text{ kJ/kg.K}$$

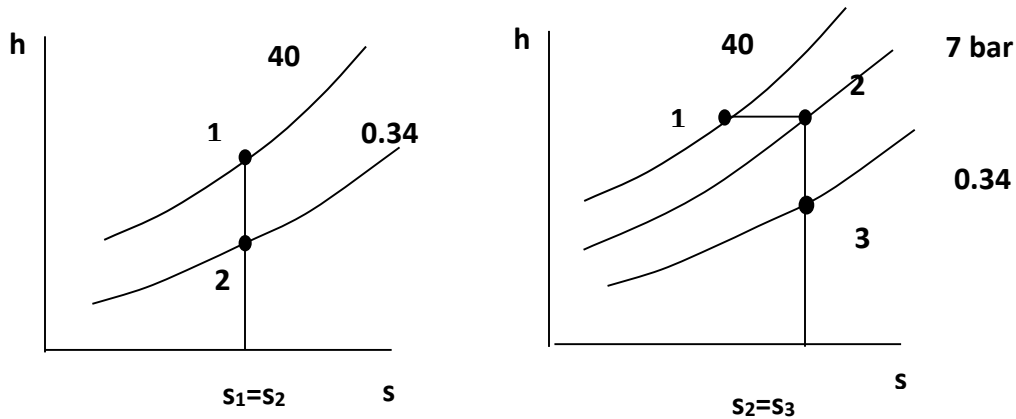
Since $s_3 < s_g$ at 0.34 bar the steam is wet at turbine exit.

$$x = \frac{s_3 - s_f}{s_g - s_f} = \frac{7.12 - 0.98}{7.73 - 0.98} = 0.909$$

$$\therefore h_3 = h_f + xh_{fg} = 301.3 + 0.909 \times 2329 = 2419.8 \text{ kJ/kg}$$

$$\therefore \dot{W}_2 = \dot{m}(h_2 - h_3) = \dot{m}(2960.7 - 2419.8) = 540.87\dot{m} \text{ kW}$$

$$\therefore \frac{\dot{W}_1 - \dot{W}_2}{\dot{W}_1} \times 100 = \frac{802.58\dot{m} - 540.87\dot{m}}{802.58\dot{m}} \times 100 = 32.6\%$$



Mixing Process

This is the mixing of two or more streams with same or different fluids at same or different states to form one stream with no chemical reaction between different streams as shown in fig (7-12). Practical examples of mixing processes are the mixing of different streams in central air conditioning systems and vapor condensers.

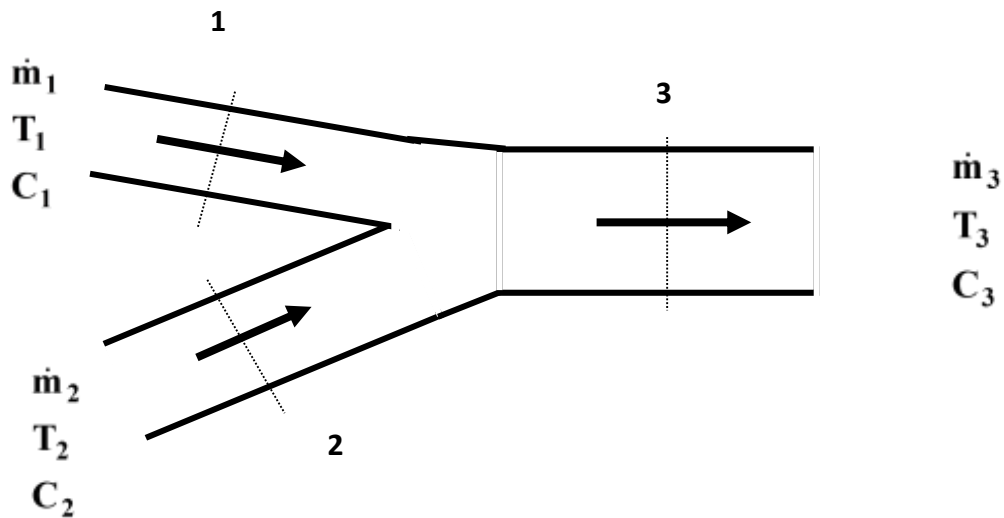


Fig (7-12) Schematic Representations of Mixing Process

Mixing process may be assumed as adiabatic process but in many cases heat transfer occurs between system and its surroundings. However the process is strongly irreversible due to eddies formed. The process is a steady flow process and the steady flow energy equation in addition to continuity equation can be used. No work is done during the mixing processes ($W=0$) and changes in KE and PE are neglected.

$$\sum \dot{m}_i = \dot{m}_e \quad (7-38)$$

$$\therefore \dot{m}_1 + \dot{m}_2 = \dot{m}_3 \quad (7-39)$$

$$\sum \dot{m}_i \left(h_i + \frac{1}{2} C_i^2 \right) + \dot{Q} = \dot{m}_e \left(h_e + \frac{1}{2} C_e^2 \right) + \dot{W} \quad (7-40)$$

By accounting for the above assumptions equation (7-40) becomes,

$$\dot{m}_1 h_1 + \dot{m}_2 h_2 + \dot{Q} = \dot{m}_3 h_3 \quad (7-41)$$

For adiabatic process,

$$\dot{m}_1 h_1 + \dot{m}_2 h_2 = \dot{m}_3 h_3 \quad (7-42)$$

For vapors the enthalpies are obtained from tables according to vapor qualities and states and if the streams are of perfect gases the gas laws and relationships are used.

$$\sum \dot{m}_i c_{p_i} T_i = \dot{m}_e c_{p_e} T_e \quad (7-43)$$

$$\therefore \dot{m}_1 c_{p_1} T_1 + \dot{m}_2 c_{p_2} T_2 = \dot{m}_3 c_{p_3} T_3 \quad (7-44)$$

For mixing of streams of same fluid c_p is omitted from both sides of the equation.

$$\dot{m}_1 T_1 + \dot{m}_2 T_2 = \dot{m}_3 T_3 \quad (7-45)$$

From equations (7-45) and (7-39) the temperature after mixing is obtained.

$$T_3 = \frac{\dot{m}_1 T_1 + \dot{m}_2 T_2}{(\dot{m}_1 + \dot{m}_2)} \quad (7-46)$$

For mixing of different fluids the specific heats at constant pressure for all inlet and exit stream must be known.

Unsteady Flow Processes

A large number of practical engineering processes are unsteady since the mass and energy within the system boundaries vary with time. Therefore such processes are sometimes referred to as transient processes. These process occur between two equilibrium states such as load reduction in steam turbine unit or gas turbine unit. This load reduction results in reduction of steam mass flow rate which induced a reduction in fuel supply and a series of reduction processes until the system reaches a new equilibrium state. Another example of transient processes is the filling or emptying of a certain reservoir. Let us consider again the continuity (7-8) and energy equation (7-19) derived previously.

$$\frac{dm_{cv}}{dt} + \sum \dot{m}_e - \sum \dot{m}_i = 0$$

$$\dot{Q} + \sum \dot{m}_i \left(h_i + \frac{C_i^2}{2} + z_i g \right) = \frac{dE_{cv}}{dt} + \sum \dot{m}_e \left(h_e + \frac{C_e^2}{2} + z_e g \right) + \dot{W}_{cv}$$

The terms $\frac{dm_{cv}}{dt}$ and $\frac{dE_{cv}}{dt}$ represent the changes in mass and energy within the control volume respectively.

These two equations are used in the thermodynamic analyses the unsteady flow processes and to simplify the analyses the following assumption is made, (the fluid properties at inlet, exit and within the control volume are uniform).

Assume that at time (t) the energy within the control volume is ($m_1 e_1$) and after time (δt) the energy within the control volume becomes ($m_2 e_2$). Therefore the energy change within the control volume during the period (δt) is,

$$(\Delta E)_{cv} = m_2 e_2 - m_1 e_1 \quad (7-47)$$

When the KE and PE of the fluid within the control volume are neglected equation (4-47) becomes,

$$(\Delta U)_{cv} = m_2 u_2 - m_1 u_1 \quad (7-48)$$

The rate of change of internal energy within the control volume during the time period (δt) is $\frac{m_2 u_2 - m_1 u_1}{\delta t}$.

When the system undergoes an unsteady flow process during the time period (δt) between two equilibrium states the energy equation for the system becomes,

$$\delta m_i \left(h_i + \frac{C_i^2}{2} + z_i g \right) + Q = (m_2 u_2 - m_1 u_1) + \delta m_e \left(h_e + \frac{C_e^2}{2} + z_e g \right) + W_{cv} \quad (7-49)$$

Where,

δm_i = mass of fluid enters the control volume during (δt),

δm_e = mass of fluid leaves the control volume during (δt),

Q = quantity of heat exchanged with surroundings during (δt),

W_{cv} = quantity of work done during (δt),

The term $\delta m_i \left(h_i + \frac{C_i^2}{2} + z_i g \right)$ represents the total energy of fluid entering the control volume and

the term $\delta m_e \left(h_e + \frac{C_e^2}{2} + z_e g \right)$ represents the total energy of fluid leaving the control volume.

Equation (7-49) is the energy equation for unsteady flow processes.

The continuity equation for unsteady flow processes is,

$$m_2 - m_1 = \delta m_i - \delta m_e \quad (7-50)$$

Where (m_1) is the mass of fluid within the control volume at the start of process and (m_2) is the mass of fluid within the control volume at the end of process.

Equation (7-50) indicates that the change of mass within the control volume during (δt) equals the difference between the mass flow in and the mass flow out during the same period of time.

Three types of unsteady flow processes is considered next.

1-Filling of an Empty Small Tank from an Infinite Reservoir

The infinite reservoir means that the thermodynamic properties of the fluid are independent of time. Example of this process is the filling of a small gas cylinder from a large tank as shown in fig (7-13). The small reservoir could be evacuated completely or partially. In the filling process there is no mass flow out of the system ($\delta m_e = 0$)

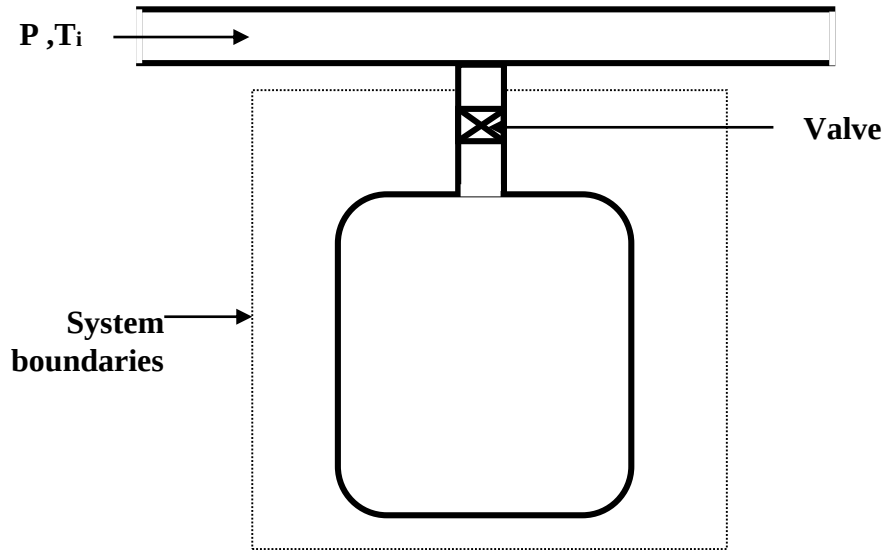


Fig (7-13) Schematic Representation of Filling Process

Consider a filling process of a small cylinder with a volume (V). The following assumptions are made.

- 1-The filling process is adiabatic ($Q=0$) since it is fast.
- 2-Kinetic and potential energies of fluid in are small and neglected.
- 3-No work is done ($W=0$).

Accounting for these assumptions (7-49) becomes,

$$m_2 u_2 - m_1 u_1 = \delta m_i h_i \quad (7-51)$$

This equation is used for all types of fluid (vapors or perfect gases).

The continuity equation (7-50) for filling process is,

$$m_2 - m_1 = \delta m_i \quad (7-52)$$

For vapors the properties are obtained from tables.

For perfect gases the following relations are used ($h = c_p T$, $u = c_v T$)

$$\therefore m_2 c_v T_2 - m_1 c_v T_1 = (m_2 - m_1) c_p T_i \quad (7-53)$$

After rearrangement equation (7-52) becomes,

$$c_v \left(\frac{m_2}{m_1} T_2 - T_1 \right) = c_p T_i \left(\frac{m_2}{m_1} - 1 \right)$$

Form equation of state of perfect gas,

$$m_2 = \frac{P_2 V}{RT_2}, \quad m_1 = \frac{P_1 V}{RT_1}$$

$$\therefore \frac{m_2}{m_1} = \frac{P_2 V}{RT_2} \times \frac{RT_1}{P_1 V} = \frac{P_2 T_1}{P_1 T_2}$$

Where,

P_1 =initial pressure inside the small cylinder,

P_2 = final pressure inside the small cylinder,

T_1 =initial temperature inside the small cylinder,

T_2 =final temperature inside the small cylinder

$$\therefore c_v \left[\frac{P_2 T_1}{P_1 T_2} \times T_2 - T_1 \right] = c_p T_i \left[\frac{P_2 T_1}{P_1 T_2} - 1 \right]$$

After simplification this equation becomes,

$$T_1 \left[\frac{P_2}{P_1} - 1 \right] = \gamma T_i \left[\frac{P_2 T_1}{P_1 T_2} - 1 \right] \quad (7-54)$$

When the final pressure P_2 equals the large reservoir pressure P the final temperature inside the small cylinder T_2 can be found from equation (7-53).

If the small cylinder is completely evacuated initially then,

$$m_1 = 0 \quad \text{and} \quad m_1 u_1 = 0$$

Hence the energy and continuity equations become,

$$m_2 u_2 = \delta m_i h_i, \quad m_2 = \delta m_i$$

$$\therefore m_2 (u_2 - h_i) = 0 \quad (7-55)$$

The final mass (m_2) cannot be zero.

$$\therefore (u_2 - h_i) = 0$$

$$\therefore u_2 = h_i$$

$$c_v T_2 = c_p T_i$$

$$\therefore T_2 = \gamma T_i \quad (7-56)$$

The mass that enters the small cylinder is calculated from the equation of state of perfect gas knowing the final pressure and temperature.

$$m_2 = \frac{P_2 V}{RT_2}$$

Emptying Process

This process is of two kinds; emptying directly to atmosphere and emptying through a turbine.

A-Emptying Directly to Atmosphere

The emptying process may be deliberate as in the case of a boiler safety valve or opening of a relief valve or accidental as in the case of leakage from a reservoir as shown schematically in fig (7-14). The state of the fluid within the reservoir before and after the process must be considered during the thermodynamic analyses of this process. Let us make the following assumptions.

1-The process is adiabatic since it is very fast ($Q=0$).

2-No work is done during the process ($W=0$).

3- No mass flow into the reservoir ($\delta m_i = 0$).

4-The KE and PE of the fluid leaving the reservoir are neglected.

After accounting for these assumptions equation (7-49) becomes,

$$(m_2 u_2 - m_1 u_1) + \delta m_e h_e = 0 \quad (7-57)$$

While the continuity equation (7-50) becomes,

$$m_1 - m_2 = \delta m_e \quad (7-58)$$

For vapors u_1, u_2, h_e are obtained from vapor tables according to its quality and state. For perfect gas the isentropic relations are used to obtain the properties of fluid remains within the reservoir assuming that it undergoes an isentropic expansion process.

$$T_2 = T_1 \left(\frac{P_2}{P_1} \right)^{\frac{\gamma-1}{\gamma}}$$

Using the relations $u = c_v T$, $h = c_p T$ and the continuity equation the energy equation becomes,

$$m_2 c_v T_2 - m_1 c_v T_1 = c_p T_2 (m_2 - m_1) \quad (7-59)$$

$$c_v T_2 \left(m_2 - m_1 \frac{T_1}{T_2} \right) = c_p T_2 (m_2 - m_1)$$

$$\therefore m_2 - m_1 \left(\frac{T_1}{T_2} \right) = \gamma (m_2 - m_1) \quad (7-60)$$

Divide both side of this equation by (m_2) ,

$$\frac{m_2}{m_1} - \frac{T_1}{T_2} = \gamma \left(\frac{m_2}{m_1} - 1 \right)$$

$$\therefore \frac{m_2}{m_1} (\gamma - 1) = \gamma - \frac{T_1}{T_2} \quad (7-61)$$

Apply the equation of state before and after the process,

$$\frac{m_2}{m_1} = \frac{P_2 V}{RT_2} \times \frac{RT_1}{P_1 V} = \left(\frac{P_2}{P_1} \right) \left(\frac{T_1}{T_2} \right)$$

$$\therefore \left(\frac{P_2}{P_1} \right) \left(\frac{T_1}{T_2} \right) (\gamma - 1) = \gamma - \frac{T_1}{T_2} \quad (7-62)$$

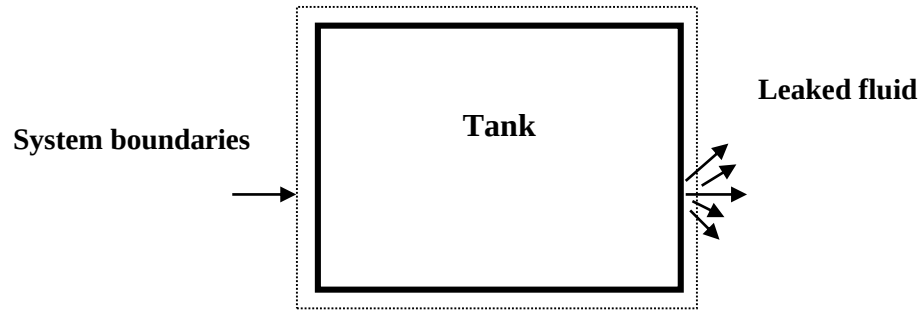


Fig (7-14) Schematic Representation of Emptying Process

B-Emptying through a Turbine

In the thermodynamic analyses the same assumption as in the case are used except the fluid is doing work as it expanded through the turbine. A schematic representation of the process is shown in fig (7-15).

$$(m_2 u_2 - m_1 u_1) + \delta m_e h_e + W = 0$$

$$\therefore W = (m_1 u_1 - m_2 u_2) - \delta m_e h_e \quad (7-63)$$

The continuity equation is,

$$m_1 - m_2 = \delta m_e$$

The same derivation procedure as in the previous case is used.

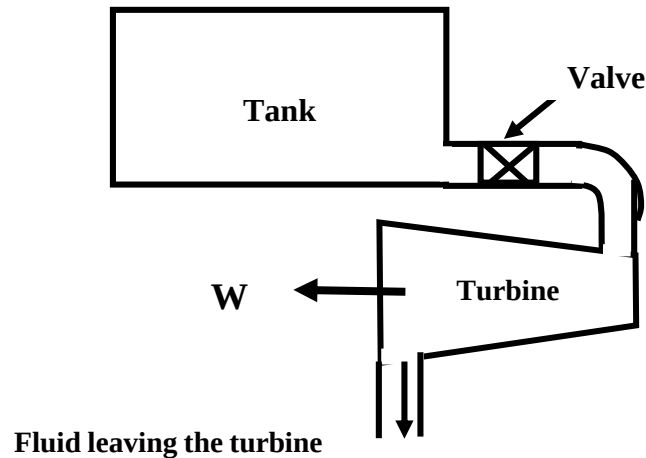


Fig (7-15) Schematic Representation of the process

Filling and Emptying Process

In this case the filling and emptying processes occur simultaneously but with different flow rates. After neglecting the KE and PE of the fluid in and out of the control volume the energy equation becomes,

$$Q + \delta m_i h_i = (m_2 u_2 - m_1 u_1) + \delta m_e h_e + W \quad (7-64)$$

$$\delta m_i \neq \delta m_e$$

The continuity equation is,

$$m_2 - m_1 = \delta m_i - \delta m_e \quad (7-65)$$

Other assumptions may be made depending on the type of fluid and its properties at start and end of process and whether work is done or not. Also whether the process is adiabatic or not.

Example 7-6

A compressed air tank is used to start an engine. The pressure and temperature inside the tank are 34 bar and 15 °C. The turbine generates a power of 4 kW during 30 seconds. Assume that the air inside the tank undergoes an isentropic expansion process to a pressure of 3.4 bar. Calculate the total volume of the tank assuming that the air leaves the turbine at 1.01 bar.

The process is an emptying process of a tank with work done.

Since the air inside the tank undergoes an isentropic process the final temperature inside the tank is calculated from the following equation.

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1} \right)^{\frac{\gamma-1}{\gamma}}$$

$$T_2 = T_1 \left(\frac{P_2}{P_1} \right)^{\frac{\gamma-1}{\gamma}} = 288 \left(\frac{3.4}{34} \right)^{\frac{1.4-1}{1.4}} = 81.896 \text{ K}$$

$$W = \dot{W} \times t = 4 \times 30 = 120 \text{ kJ}$$

$$\frac{m_2}{m_1} = \frac{P_2 T_1}{P_1 T_2} = \frac{3.4 \times 288}{34 \times 81.896} = 0.3517$$

$$W = (m_1 u_1 - m_2 u_2) - (m_1 - m_2) h_2$$

$$W = m_1 c_v \left(T_1 - \frac{m_2}{m_1} T_2 \right) - m_1 c_p T_2 \left(1 - \frac{m_2}{m_1} \right)$$

$$120 = 0.718 \times m_1 (288 - 0.3517 \times 81.896) - 1.005 \times 81.896 \times m_1 (1 - 0.3517)$$

$$\therefore m_1 = 0.904 \text{ kg}$$

$$V = \frac{m_1 R T_1}{P_1} = \frac{0.904 \times 287 \times 288}{34 \times 10^5} = 0.0219 \text{ m}^3$$