

FIRST LAW OF THERMODYNAMICS AND NON FLOW PROCESSES

Prof. Haroun A K Shahad

The non-flow processes occur in the closed system and classified into two types; either reversible non-flow processes or irreversible non-flow processes. The first law of thermodynamics is valid for both types and the resulting equation is known as the non-flow energy equation as mentioned in the previous lecture. All actual non-flow processes are irreversible while the reversible process is an ideal process.

According to the first corollary of the first law of thermodynamics the change in internal or intrinsic energy of a system during a process equals the difference between work done and heat supplied.

$$Q - W = E_2 - E_1 \quad (6-1)$$

OR,

$$Q - W = U_2 - U_1 \quad (6-2)$$

$$U = mu \quad (6-3)$$

where,

m=mass of system (kg)

u=specific internal energy of system (kJ/kg)

Equation (6-2) can be written in differential form as,

$$\delta Q - \delta W = dU \quad (6-4)$$

OR per unit mass as,

$$\delta q - \delta w = du \quad (6-5)$$

All the above equations can be used for reversible and irreversible processes.

For a reversible process the work can be expressed as,

$$\delta w = P dv \quad (6-6)$$

$$\therefore \delta q = du + P dv \quad (6-7)$$

Example 6-1

A closed tube with a constant volume of 100 mL contains a mixture of saturated liquid water and saturated steam in equilibrium at 100 kPa in a proportion that makes the mixture passes through the critical state when heated. Calculate the amount of heat required to make the steam reaches the critical state.

Since the tube is closed the system is a closed system and the non-flow energy equation can be used.

Since the volume of tube is constant no work is done during the heating process ($W=0$).

$$Q - W = U_2 - U_1$$

$$\therefore Q = U_2 - U_1 = m(u_2 - u_1)$$

Since the mixture reaches through the critical state at the end of heating its properties equal the properties of critical state which can be obtained from steam tables.

$$\therefore u_2 = u_c, v_2 = v_c$$

$$u_2 = 2038 \text{ kJ/kg}, v_2 = 0.00317 \text{ m}^3/\text{kg}$$

Initially there is a mixture of saturated liquid and saturated steam.

$$u_1 = u_{f1} + x_1(u_{g1} - u_{f1})$$

$$v_1 = x_1 v_{g1}$$

Since the tube is closed and has a constant volume the mixture specific volume remains constant during heating process.

$$v_1 = v_2 = x_1 v_{g1}$$

Find v_g at 100 kPa from table.

$$v_{g1} = 1.694 \text{ m}^3/\text{kg}$$

$$\therefore x_1 = \frac{v_2}{v_{g1}} = \frac{0.00317}{1.694} = 0.00187$$

$$m = \frac{V}{v_1} = \frac{100 \times 10^{-6}}{0.00317} = 0.0315 \text{ kg}$$

$$u_1 = 417 + 0.00187(2506 - 417) = 421 \text{ kJ/kg}$$

$$\therefore Q = m(u_2 - u_1) = 0.0315(2038 - 421) = 50.9 \text{ kJ}$$

Specific Heats

The specific heat is defined as “the amount of heat required to raise the temperature of a unit mass of system one degree temperature rise”. The amount of heat added during a process depends on the path of the process and since there are infinite possible paths between any two states there would be infinite possible specific heats. In this context we will consider only two important specific heats which are specific heat at constant volume and specific heat at constant pressure (c_p).

Specific Heat at Constant Volume

It is defined as follows “the amount of heat required to raise the temperature of a unit mass of system one degree temperature rise when the heat added at constant volume ($\Delta V = 0$)”. It is given the symbol (c_v) per unit mass and (C_v) per unit mole.

There is another definition for the specific heat at constant volume which is stated as “the change in internal energy of a unit mass or mole of the system when its temperature varies by one degree temperature between two states of the same volume”. The relation between the mass specific heat and molar specific heat is,

$$C_v = mw \times c_v \quad (6-8)$$

Where mw is the molecular weight of system.

Since the process is a constant volume process then ($\Delta V = 0$) and the non-flow energy equation becomes,

$$q = u_2 - u_1 = \Delta u \quad (6-9)$$

According to the definition of specific heat it can be expressed mathematically as,

$$c_v = \left[\frac{q}{\Delta T} \right]_v \quad (6-10)$$

$$\therefore c_v = \left(\frac{q}{\Delta T} \right)_v = \left(\frac{\Delta u}{\Delta T} \right)_v \quad (6-11)$$

The term $\left(\frac{\Delta u}{\Delta T} \right)_v$ represents the average specific at constant volume through the range of temperature ΔT . The average specific heat at constant volume varies with ΔT and specific volume. As ΔT approaches zero the value of term $\left(\frac{\Delta u}{\Delta T} \right)_v$ approaches $\left(\frac{\partial u}{\partial T} \right)_v$. Hence we can write,

$$c_v = \left(\frac{\partial u}{\partial T} \right)_v \quad (6-12)$$

As the internal energy is a property of system it can be expressed in terms of any other two properties of the system.

$$u = f(T, v) \quad (6-13)$$

$$\therefore du = \left(\frac{\partial u}{\partial T} \right)_v dT + \left(\frac{\partial u}{\partial v} \right)_T dv \quad (6-14)$$

For constant volume process ($dv=0$) hence,

$$\therefore du = \left(\frac{\partial u}{\partial T} \right)_v dT \quad (6-15)$$

$$\therefore du = c_v dT \quad (6-16)$$

Equation (6-16) is similar to Joules law for perfect gases which is state as follows “the internal energy of a perfect gas is a function of temperature only” which is expressed mathematically as,

$$u = f(T) \quad (6-17)$$

This law was proved experimentally by the scientist Joule in 1843 as follows.

Consider two closed vessel (A and B) connected by a pipe through valve as shown in fig (6-1).

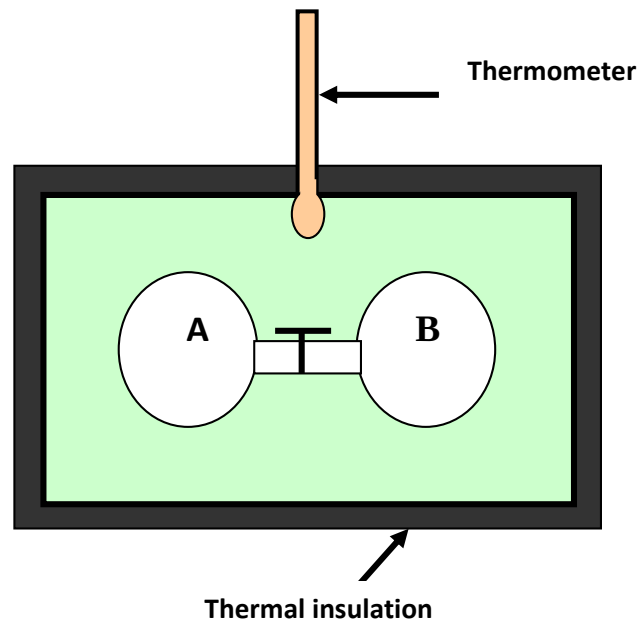


Fig (6-1)

One of the vessels contains air at 22 atm. while the other is evacuated. The two vessels are immersed in a thermally insulated water bath. When the vessels and water bath reaches thermal equilibrium the valve is opened so that the air expands to fill both vessels until the pressure is the same in both vessels. It was found that the temperature of the water bath remains constant which means no heat exchange occurs between the water bath and the air in both vessels ($Q=0$). Also no work is done during the process ($W=0$).

Apply the non-flow energy equation,

$$Q - W = U_2 - U_1$$

$$\therefore U_2 = U_1$$

Since the pressure and volume of air are changed during the process while the internal energy remains constant Joule concluded that the internal energy is independent of pressure and volume but depends on temperature only.

To find the functional relationship of equation (6-16) assume a unit mass of a perfect gas is heated at constant volume. Apply the non-flow energy equation in differential form for this process.

$$\delta q = du + \delta w$$

But, $\delta w = 0$ since the volume is constant.

$$\therefore \delta q = du$$

From equation (6-16),

$$du = c_v dT$$

Since c_v is constant for perfect gas,

$$\int du = \int c_v dT = c_v \int dT$$

$$\therefore u = c_v T + k \quad , \text{ where } k \text{ is the constant of integration.}$$

Since the internal energy of a perfect gas equals zero at absolute zero the constant of integration (k) equals zero.

$$\therefore u = c_v T \tag{6-17}$$

If a perfect gas undergoes a process between two states (1) and (2) the change of internal energy of the gas during the process is,

$$u_2 - u_1 = c_v T_2 - c_v T_1 = c_v (T_2 - T_1)$$

Since $\delta q = du$ for constant volume process,

$$\int_1^2 \delta q = \int_1^2 du$$

$$\therefore q_v = \int_1^2 c_v dT = c_v (T_2 - T_1)$$

Where (q_v) represents the heat exchanged during the constant pressure process. For a mass (m) the total heat exchanged equals,

$$Q_v = mc_v (T_2 - T_1)$$

Specific Heat at Constant Pressure

It is defined as follows “the amount of heat required to raise the temperature of a unit mass of system one degree temperature rise when the heat added at constant pressure ($\Delta P = 0$)”. It is given the symbol (c_p) per unit mass and (C_p) per unit mole.

There is another definition for the specific heat at constant pressure which is stated as “the change in enthalpy of a unit mass or mole of the system when its temperature varies by one degree temperature between two states of the same pressure”. The relation between the mass specific heat and molar specific heat is,

$$C_p = mwc_p$$

Where mw is the molecular weight of pure substance.

The enthalpy is another property of pure the pure substance which depends on the internal energy, volume and pressure and given the symbol (h). It is expressed mathematically as,

$$h = u + Pv \tag{6-18}$$

By differentiating the above equation at constant pressure,

$$dh = du + d(Pv) \tag{6-19}$$

The total enthalpy is given as,

$$H = mh = m(u + Pv) = U + PV \tag{6-20}$$

The units of specific enthalpy (h) is (J/kg) and the units of total enthalpy (H) is (J).

Apply the non-flow energy equation in differential form,

$$\delta q = du + Pd v$$

Since the pressure is constant during the process,

$$\delta q = du + d(Pv) \tag{6-21}$$

From the comparison of equations (6-19) and (6-21) we find that,

$$\delta q = dh \quad (6-22)$$

This means that the amount of heat supplied equals the change in specific enthalpy during the process.

From the definition of specific heat at constant pressure we can write,

$$c_p = \left[\frac{q}{\Delta T} \right]_p$$
$$\therefore \left(\frac{q}{\Delta T} \right)_p = \left(\frac{\Delta h}{\Delta T} \right)_p \quad (6-23)$$

The term $\left(\frac{\Delta h}{\Delta T} \right)_p$ represents the mean specific heat at constant pressure through the temperature range ΔT . As ΔT approaches zero the term $\left(\frac{\Delta h}{\Delta T} \right)_p$ approaches $\left(\frac{\partial h}{\partial T} \right)_p$ which represents the specific heat at constant pressure.

$$\therefore c_p = \left(\frac{\partial h}{\partial T} \right)_p \quad (6-24)$$

As the enthalpy is a property of system it can be described by any other two properties for example pressure and temperature.

$$h = f(T, P) \quad (6-25)$$

$$dh = \left(\frac{\partial h}{\partial T} \right)_p dT + \left(\frac{\partial h}{\partial P} \right)_T dP \quad (6-26)$$

Since the process is at constant pressure ($dP = 0$),

$$\therefore dh = \left(\frac{\partial h}{\partial T} \right)_p dT \quad (6-27)$$

From equations (6-24) and (6-27) we find,

$$dh = c_p dT \quad (6-28)$$

For perfect gas substitute for (dh) from equation (6-22), hence,

$$\delta q = c_p dT \quad (6-29)$$

By integrating equation (6-29) between states (1) and (2), the heat exchanged with the surroundings is obtained.

$$\int_1^2 \delta q = \int_1^2 c_p dT \quad (6-30)$$

Now for constant c_p ,

$$\therefore q_p = c_p (T_2 - T_1) \quad (6-31)$$

For mass of system (m),

$$Q_p = mc_p (T_2 - T_1) \quad (6-32)$$

The heat exchanged for vapors is obtained by integrating equation (6-22) between states (1) and (2).

$$\int_1^2 \delta q = \int_1^2 dh$$

$$\therefore q = h_2 - h_1 \quad (6-33)$$

For the solid and liquid phases of the pure substance the specific heats c_p, c_v are approximately equal since the coefficient of volumetric expansion is very small and can be neglected.

$$\left(\frac{\partial v}{\partial T} \right)_p \cong 0 \quad (6-33)$$

Therefore the heat transfer process at constant pressure can be considered as a heat transfer at constant volume i e,

$$\left(\frac{\partial h}{\partial T} \right)_p \cong \left(\frac{\partial h}{\partial T} \right)_v \quad (6-34)$$

$$dh = du + d(Pv)$$

Since the pressure is constant,

$$\therefore dh = du + P(dv)$$

However, $(dv \cong 0)$,

$$\therefore dh = du$$

$$\therefore \left(\frac{\partial h}{\partial T} \right)_p \cong \left(\frac{\partial h}{\partial T} \right)_v = \left(\frac{\partial u}{\partial T} \right)_v$$

$$\therefore c_p \cong c_v = c \quad (6-35)$$

In the gas phase of the pure substance there is a clear difference between the values of c_p, c_v since the volume varies with pressure and temperature. Many mathematical relations can be derived relating these specific heats as shown below.

Assume a mass (m) of a perfect gas is heated from T_1 to T_2 at constant pressure. Apply the non-flow energy equation for this process.

$$Q = (U_2 - U_1) + W$$

$$\therefore U_2 - U_1 = mc_v(T_2 - T_1) \quad , W = P(V_2 - V_1) = mR(T_2 - T_1)$$

$$\therefore Q = mc_v(T_2 - T_1) + mR(T_2 - T_1) \quad (6-36)$$

From equation (6-32),

$$Q = mc_p(T_2 - T_1)$$

Equating equations (6-32) and (6-36),

$$mc_p(T_2 - T_1) = mc_v(T_2 - T_1) + mR(T_2 - T_1)$$

Hence,

$$\therefore c_p = c_v + R \quad \text{OR}$$

$$R = c_p - c_v \quad (6-37)$$

Multiply both sides of equation (6-37) by the molecular weight (mw) of the gas,

$$mwR = mwc_p - mwc_v$$

$$\therefore R_o = C_p - C_v \quad (6-38)$$

Where (R_o) is the universal gas constant and (R) is the specific gas constant which is always positive, hence $c_p > c_v$ for all gases.

The ratio between the two specific heats (γ) is known as the adiabatic index and used in many mathematical thermodynamic derivations.

$$\gamma = \frac{c_p}{c_v} \quad (6-39)$$

For all gases ($\gamma > 1.0$).

For monatomic gases $\gamma = 1.6$.

For diatomic gases $\gamma = 1.4$.

For triatomic gases $\gamma = 1.33$.

By dividing equation (6-37) by (c_v) the following relation is obtained.

$$\frac{R}{c_v} = \gamma - 1$$
$$\therefore c_v = \frac{R}{\gamma - 1} \quad (6-40)$$

From equation (6-39) and by substituting in equation (6-40) with rearrangement the following equation is obtained.

$$c_p = \frac{\gamma R}{\gamma - 1} \quad (6-41)$$

THE REVERSIBLE NON-FLOW PROCESSES

There are many possible non-flow processes undergone by a system but we will consider only five processes which are important.

1-The constant volume process

2-The constant pressure process

3-The constant temperature process

4-The adiabatic process

5-The polytropic process

In the following sections we will discuss these processes in more details with reference to the application of the non-flow energy equation for each process.

1-The Constant Volume Process

It is the process during which the system boundaries are fixed and not moving so that the system volume remains constant. It is also referred to as the **isochoric process**. Since the system volume is constant no work is done during this process neither by the system on surroundings nor by surroundings on the system. There only one possible constant volume process where work is done by surroundings on the system which is the mixing process by a mixer. This type of work is known as peddling work. This process is represented by a vertical line on the P-V diagram as shown in fig (6-2) for vapor and perfect gas.

Apply the non-flow energy equation for this process,

$$Q = (U_2 - U_1) + W$$

$$W = 0$$

$$\therefore Q = U_2 - U_1 = m(u_2 - u_1)$$

If the working fluid is vapor u_1 and u_2 are obtained from tables and if the working fluid is perfect gas apply Joules law. The non-flow energy equation for perfect gas becomes,

$$Q = mc_v(T_2 - T_1)$$

The equation of state for perfect gas becomes,

$$\frac{P_1}{T_1} = \frac{P_2}{T_2} \quad (6-42)$$

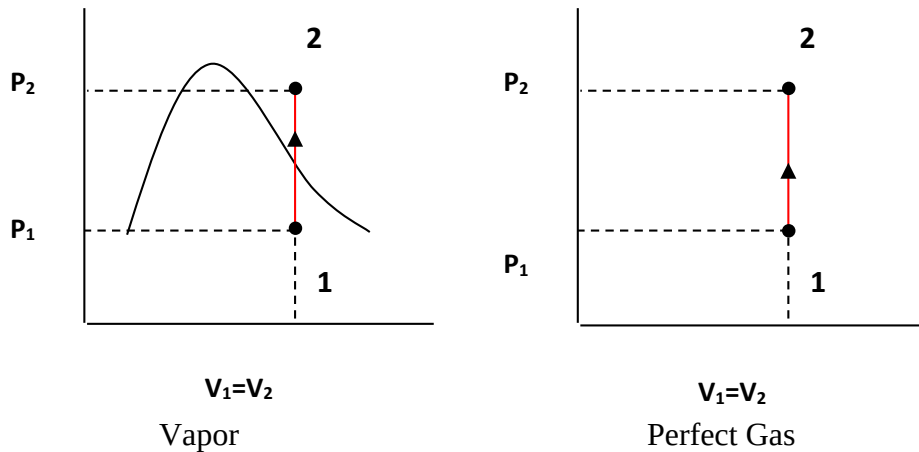


Fig (6-2) Constant Volume Process on P-V Diagram

Example 6-2

A solid cylinder with a volume of 0.025 m^3 contains steam at 80 bar and 350°C . The cylinder is cooled until the pressure becomes 50 bar. Determine the quality of steam and the amount of heat lost at the end of the process.

Since the cylinder is solid its volume remains constant during the cooling process which is a non-flow process. Apply the non-flow energy equation.

$$Q = (U_2 - U_1) + W = m(u_2 - u_1) + W$$

Since the volume is constant $W = 0$

$$\therefore Q = m(u_2 - u_1)$$

From steam tables we find that at 80 bar and 350°C the steam is superheated. Hence,

$$u_1 = 2750 \text{ kJ/kg} \quad v_1 = 0.02994 \text{ m}^3/\text{kg}$$

Assume that the mass of steam within the cylinder is (m).

$$m = \frac{V}{v} = \frac{0.025}{0.02994} = 0.835 \text{ kg}$$

Since the process is a constant volume process,

$$\therefore v_2 = v_1 = 0.0294 \text{ m}^3 / \text{kg}$$

From steam tables the specific volume of saturated steam at 50 bar is,

$$v_g = 0.03944 \text{ m}^3 / \text{kg}$$

Since $v_2 < v_g$ the steam at the end of cooling process is wet.

$$x = \frac{v_2}{v_g} = \frac{0.02994}{0.03944} = 0.758$$

$$u_2 = u_f + x(u_g - u_f)$$

Also we find u_f and u_g at 50 bar from steam tables,

$$\therefore u_2 = 1149 + 0.758(2597 - 1149) = 2247 \text{ kJ/kg}$$

After substitution in the non-flow energy equation,

$$Q = 0.835(2247 - 2750) = -420 \text{ kJ}$$

which is the heat lost from the cylinder during the cooling process.

Example 6-3

A solid cylinder with a volume of 0.5 m^3 contains air at 1 bar and 25°C . The cylinder is heated until the pressure becomes 5 bar. Determine the air temperature and the amount of heat supplied at the end of the process.

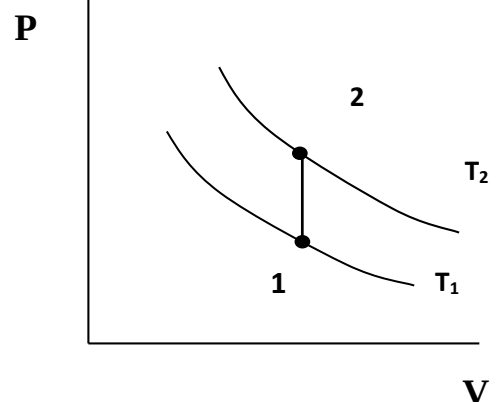
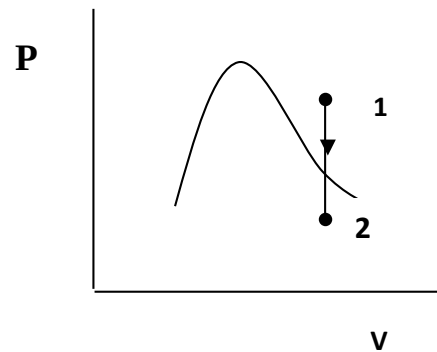
Assume that the mass of air inside the cylinder is (m).

$$m = \frac{P_1 V_1}{RT_1} = \frac{1 \times 10^5 \times 0.5}{287 \times 298} = 0.585 \text{ kg}$$

Since the cylinder volume is constant ($V_2 = V_1$),

$$\therefore T_2 = \frac{P_2 V_2}{mR} = \frac{5 \times 10^5 \times 0.5}{0.585 \times 287} = 1489 \text{ K}$$

Apply the non-flow energy equation,



$$Q = m(u_2 - u_1) + W$$

Since the volume is constant no work is done,

$$\therefore Q = m(u_2 - u_1) = mc_v(T_2 - T_1)$$

$$\therefore Q = 0.585 \times 0.718 \times (1489 - 298) = 500.27 \text{ kJ}$$

2-The Constant Pressure Process

During this process the pressure remains constant therefore it is also referred to as the **isobaric process**. As the system boundaries are moving during this process the system volume is changed and work is done. The amount of work done during the process is calculated by integrating the equation of work during the process.

$$W = \int_1^2 P dV$$

Since the pressure is constant this equation can be easily integrated.

$$\therefore W = P \int_1^2 dv = P(V_2 - V_1) \quad (6-43)$$

Apply the non-flow energy equation,

$$Q = m(u_2 - u_1) + W$$

Substitute equation (6-43) for work,

After rearrangement the above equation becomes,

$$Q = m[(u_2 + Pv_2) - (u_1 + Pv_1)]$$

But $h = u + Pv$

$$\therefore Q = m(h_2 - h_1) \quad (6-44)$$

This means that the amount of heat exchanged with the system during the constant pressure process equals the change in its enthalpy during the process.

For vapor the enthalpies (h_1) and (h_2) are obtained from tables.

For perfect gas,

$$W = P(V_2 - V_1) = mR(T_2 - T_1)$$

Substitute for work in the non-flow energy equation.

$$\therefore Q = mc_v(T_2 - T_1) + mR(T_2 - T_1)$$

$$Q = m(c_v + R)(T_2 - T_1)$$

$$\therefore Q = mc_p(T_2 - T_1) \quad (6-45)$$

The equation of state for perfect gas becomes,

$$\frac{V_1}{T_1} = \frac{V_2}{T_2} \quad , \quad \frac{v_1}{T_1} = \frac{v_2}{T_2} \quad (6-46)$$

This process is represented by a horizontal line on the P-V diagram as shown in fig (6-3). The area under this line represents the work done during the process.

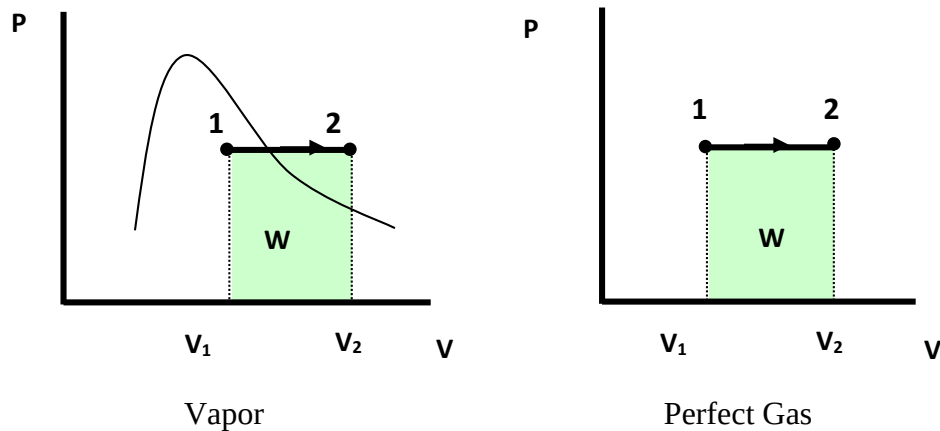


Fig (6-3) The Constant Pressure Process on P-V Diagram

Example 6-4

A quantity of a certain fluid with a mass of 0.05 kg is heated at constant pressure of 2 bar until its volume becomes 0.0658 m³. Calculate the amount of heat supplied and work done during the process if,

- 1-The fluid is saturated steam initially,
- 2-Air at a temperature of 130 °C initially.

1-The fluid is saturated steam initially.

The initial properties of steam are obtained from saturated steam table.

$$h_1 = h_g \quad , \quad v_1 = v_g$$

$$h_1 = 2707 \text{ kJ/kg} \quad , \quad v_1 = 0.8856 \text{ m}^3/\text{kg}$$

Since the process is a constant pressure non-flow process,

$$Q = m(h_2 - h_1)$$

$$W = mP(v_2 - v_1)$$

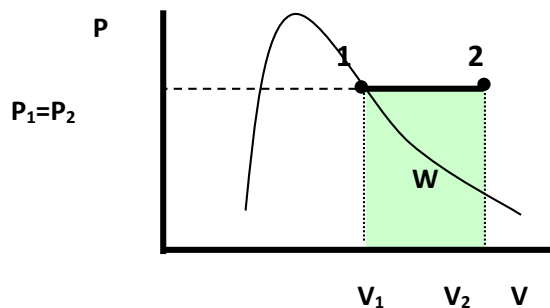
$$v_2 = \frac{V_2}{m} = \frac{0.0658}{0.05} = 1.316 \text{ m}^3/\text{kg}$$

Since $v_2 > v_g$ at 2 bar the steam is finally superheated and its enthalpy (h_2) is obtained from superheated steam tables.

$$h_2 = 3072 \text{ kJ/kg}$$

$$\therefore Q = 0.05 \times (3072 - 2707) = 18.25 \text{ kJ}$$

$$W = 0.05 \times 2 \times 10^5 \times (1.316 - 0.8856) = 4304 \text{ J} = 4.304 \text{ kJ}$$



2-The fluid is air

The process is a constant pressure non-flow process.

$$P_2 = P_1$$

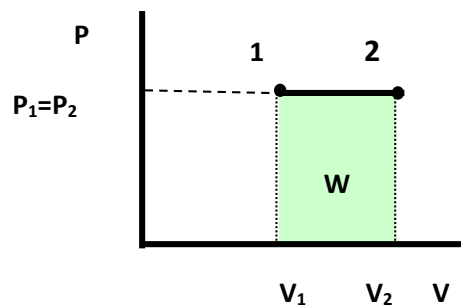
$$P_2 V_2 = mRT_2$$

$$T_2 = \frac{P_2 V_2}{mR} = \frac{2 \times 10^5 \times 0.0658}{0.05 \times 287} = 917 \text{ K}$$

$$\therefore Q = mc_p (T_2 - T_1)$$

$$= 0.05 \times 1.005 \times (917 - 403) = 25.83 \text{ kJ}$$

$$W = mR(T_2 - T_1) = 0.05 \times 0.287 \times (917 - 403) = 7.38 \text{ kJ}$$



3-The Constant Temperature Process

This process is also referred to as the **isothermal process** since the temperature at the start and end of process is the same. It is difficult to keep the temperature constant during any expansion or compression process. Therefore the system must be heated during expansion or cooled during compression to keep the temperature constant. Fig (6-5) shows this process on P-V diagram.

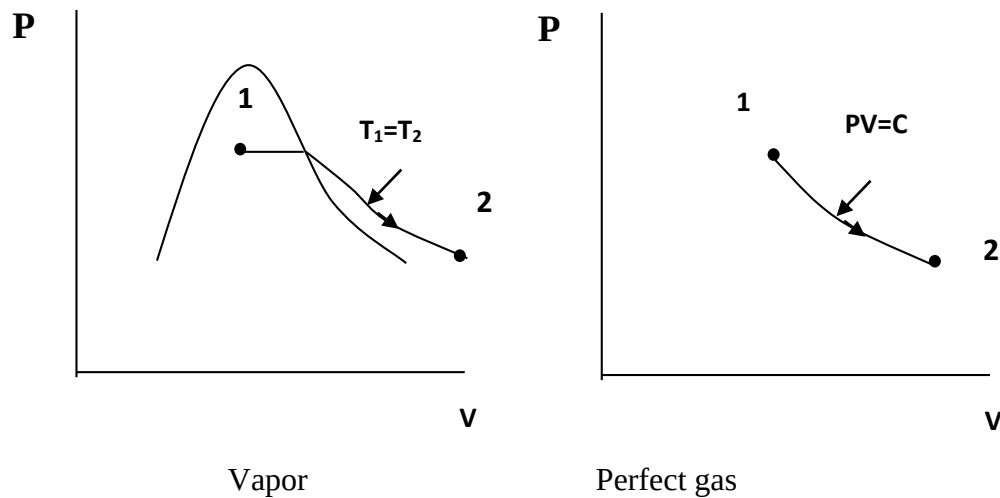


Fig (6-4) The constant temperature process on P-V diagram

Line 1-2 on the P-V diagram is known as the isothermal line or the isotherm.

Since it is a non-flow process the non-flow energy equation can be applied.

$$Q = m(u_2 - u_1) + W$$

For vapor u_1 and u_2 are obtained from tables according to quality of vapor while the work is obtained by the following equation.

$$W = \int_1^2 P dV$$

The integration of the above equation depends on the relation between pressure and volume during the process.

For perfect gas the following equations are used; Boyle's law, Joules law, equation of state, the non-flow energy equation in addition to the above equation of work.

$$PV = C \quad , \quad P_1 V_1 = P_2 V_2$$

$$Pv = RT \quad , \quad PV = mRT$$

$$u = c_v T$$

$$W = \int_1^2 P dV$$

This equation is integrated with the help of Boyles law,

$$PV = C, \quad P = \frac{C}{V}$$

$$\therefore W = \int_1^2 \frac{C}{V} dV = C \int_1^2 \frac{dV}{V}$$

$$\therefore W = C \ln \frac{V_2}{V_1}$$

$$\text{But } C = P_1 V_1 = P_2 V_2$$

$$\therefore W = P_1 V_1 \ln \frac{V_2}{V_1} = P_2 V_2 \ln \frac{V_2}{V_1} \quad (6-47)$$

From Boyles law,

$$\frac{V_2}{V_1} = \frac{P_1}{P_2}$$

$$\therefore W = P_1 V_1 \ln \frac{P_1}{P_2} = P_2 V_2 \ln \frac{P_1}{P_2} \quad (6-48)$$

Also from the equation of state,

$$P_1 V_1 = mRT_1 \quad , \quad P_2 V_2 = mRT_2$$

Since the temperature is constant ($T_1 = T_2$)

$$\therefore W = mRT \ln \frac{V_2}{V_1} = mRT \ln \frac{P_1}{P_2} \quad (6-49)$$

According to Joules law,

$$u_2 - u_1 = c_v (T_2 - T_1)$$

Since $T_1 = T_2$

$$\therefore u_2 - u_1 = 0$$

Therefore the change of internal energy of the perfect gas during the isothermal process is zero.
Hence for perfect gas,

$$Q = W \quad (6-50)$$

This means that, for perfect gas, the work done during the isothermal process equals the heat exchanged during the process.

Example 6-5

Steam at 7 bar with a dryness fraction of 0.9 expands inside a cylinder to 3 bar according to the law $PV=C$. Calculate the work done and the heat exchanged per unit mass of steam during the process.

This is an isothermal non-flow process follows the law $PV=C$.

Find the properties of the wet steam initially.

$$u_1 = u_f + x(u_g - u_f)$$

$$u_1 = 696 + 0.9 \times (2571 - 696) = 2383.5 \text{ kJ/kg}$$

$$v_1 = xv_g = 0.9 \times 0.273 = 0.2457 \text{ m}^3/\text{kg}$$

From the law $PV=C$.

$$\therefore v_2 = \frac{P_1 v_1}{P_2}$$

$$v_2 = \frac{7 \times 0.2457}{3} = 0.5733 \text{ m}^3/\text{kg}$$

From saturated steam table $v_g = 0.606 \text{ m}^3/\text{kg}$ at 3 bar.

Since $v_2 < v_g$ at 3 bar the steam remains wet at the end of process.

$$x = \frac{v_2}{v_g} = \frac{0.5733}{0.606} = 0.946$$

$$u_2 = u_f + x(u_g - u_f)$$

$$u_2 = 561 + 0.946 \times (2543 - 561) = 2436 \text{ kJ/kg}$$

$$w = P_1 v_1 \ln \frac{v_2}{v_1} = 7 \times 10^5 \times 0.2457 \ln \frac{0.5733}{0.2457} = 145726 \text{ J/kg}$$

$$= 145.726 \text{ kJ/kg}$$

$$q = (u_2 - u_1) + w$$

$$\therefore q = (2436 - 23835) + 145.726 = 198.226 \text{ kJ/kg}$$

This heat is supplied to the system since it is positive.

Example 6-6

Calculate the work done during an isothermal compression process of 0.5 mole of oxygen gas initially at 27 °C from 1 bar to 100 bar.

$$W = nR_o T \ln \frac{P_1}{P_2}$$

$$T = 27 + 273 = 300 \text{ K}$$

$$W = 0.5 \times 8314 \times 300 \ln \frac{1}{100} = -5744 \text{ J}$$

4-The Adiabatic Process

The adiabatic process is the process during which no heat is exchanged with the surroundings which means that the system is thermally isolated from its surroundings. Adiabatic processes could be reversible or irreversible. Fig (6-5) shows this process on the P-V diagram. When the adiabatic process is reversible it is referred to as **isentropic process** which means there is no change in system entropy during the process. Apply the non-flow energy equation,

$$Q = m(u_2 - u_1) + W$$

Since the process is adiabatic ($Q = 0$),

$$\therefore W = m(u_1 - u_2) \quad (6-51)$$

Equation (6-51) is used for adiabatic reversible and irreversible processes.

When the working fluid is vapor u_1 and u_2 are obtained from vapor tables according to vapor quality and the work is calculated directly from equation (6-51).

When the working fluid is perfect gas a mathematical formula is derived depending on the equation of state, non-flow energy equation and the adiabatic law as shown below.

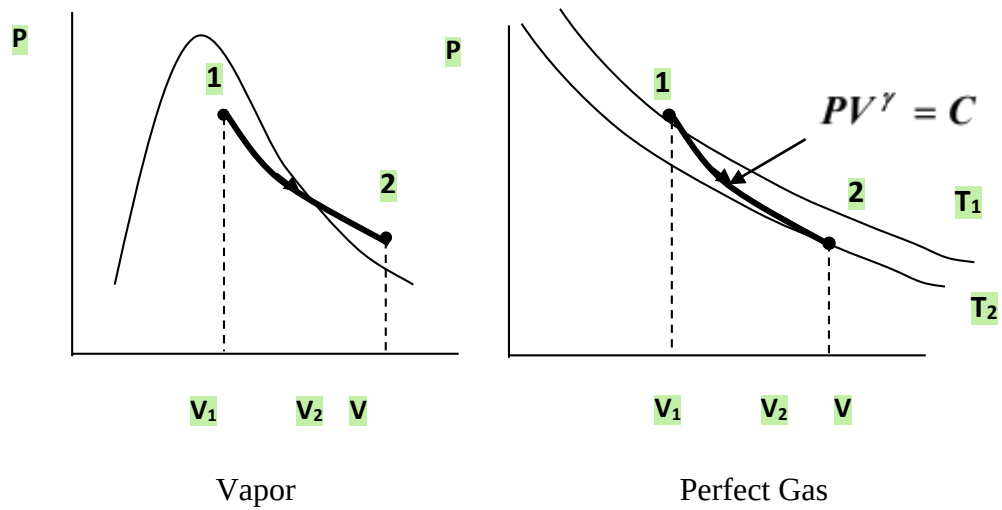


Fig (6-5) The Adiabatic Process

Let us first derive the adiabatic law starting with the non-flow energy equation in differential form.

$$\delta q = du + \delta w$$

Since, $\delta Q = 0$,

$$\therefore du + \delta w = 0$$

$$\delta w = Pdv \quad , \quad Pv = RT \quad , \quad du = c_v dT$$

$$\therefore c_v dT + RT \frac{dv}{v} = 0$$

Divide by (T),

$$c_v \frac{dT}{T} + R \frac{dv}{v} = 0$$

By integrating the above equation,

$$c_v \ln T + R \ln v = C_1$$

Where C_1 is the constant of integration.

From equation the equation of state of perfect gas,

$$T = \frac{Pv}{R}$$

$$\therefore c_v \ln\left(\frac{Pv}{R}\right) + R \ln v = C_1$$

Divide the above equation by c_v

$$\ln\left(\frac{Pv}{R}\right) + \frac{R}{c_v} \ln v = \frac{C_1}{c_v} = C_2$$

$$\frac{R}{c_v} = \gamma - 1$$

$$\therefore \ln\left(\frac{Pv}{R}\right) + (\gamma - 1) \ln v = C_2$$

$$\ln\left(\frac{Pv}{R}\right) + \ln(v)^{\gamma-1} = C_2$$

$$\ln\left(\frac{Pv^\gamma}{R}\right) = C_2$$

$$\therefore \frac{Pv^\gamma}{R} = e^{C_2}$$

$$Pv^\gamma = R \times e^{C_2} = C_3 = \text{constant}$$

$$\therefore Pv^\gamma = C \quad (6-52)$$

This equation is known as the adiabatic law of perfect gases

$$P_1 v_1^\gamma = P_2 v_2^\gamma = C$$

Where C, C_1 , C_2 and C_3 are all constants.

$$\therefore \frac{P_1}{P_2} = \left(\frac{v_2}{v_1}\right)^\gamma \quad (6-53)$$

The value of the adiabatic index (γ) depends on the type of gas. For example (γ) for air is 1.4.

Depending on equation of state of perfect gas and equation (6-53) other adiabatic ratios can be derived as follows.

$$P = \frac{RT}{v}$$

$$\therefore \frac{RT}{v} v^\gamma = C$$

$$T v^{\gamma-1} = \frac{C}{R} = C_1$$

$$\therefore T_1 v_1^{\gamma-1} = T_2 v_2^{\gamma-1} = \text{constant}$$

$$\therefore \frac{T_1}{T_2} = \left(\frac{v_2}{v_1} \right)^{\gamma-1} \quad (6-54)$$

Also from the equation of state of perfect gas,

$$v = \frac{RT}{P}$$

$$P \left(\frac{RT}{P} \right)^\gamma = C$$

$$\frac{(RT)^\gamma}{P^{\gamma-1}} = C$$

$$\frac{T^\gamma}{P^{\gamma-1}} = \frac{C}{R^\gamma} = C_1$$

$$\therefore \frac{T}{(P)^{\frac{\gamma-1}{\gamma}}} = (C_1)^{\frac{1}{\gamma}} = \text{constant}$$

$$\frac{T_1}{(P_1)^{\frac{\gamma-1}{\gamma}}} = \frac{T_2}{(P_2)^{\frac{\gamma-1}{\gamma}}} = \text{constant}$$

$$\therefore \frac{T_1}{T} = \left(\frac{P_1}{P_2} \right)^{\frac{\gamma-1}{\gamma}} \quad (6-55)$$

The equation of the adiabatic work is derived as follows.

$$W = m(u_1 - u_2) = mc_v(T_1 - T_2)$$

$$c_v = \frac{R}{\gamma - 1}$$

$$\therefore W = \frac{mR(T_1 - T_2)}{\gamma - 1}$$

From equation of state $PV = mRT$

$$\therefore W = \frac{P_1V_1 - P_2V_2}{\gamma - 1} \quad (6-56)$$

This is the equation for the work done during the adiabatic process which is represented by area (W) in fig (6-6).

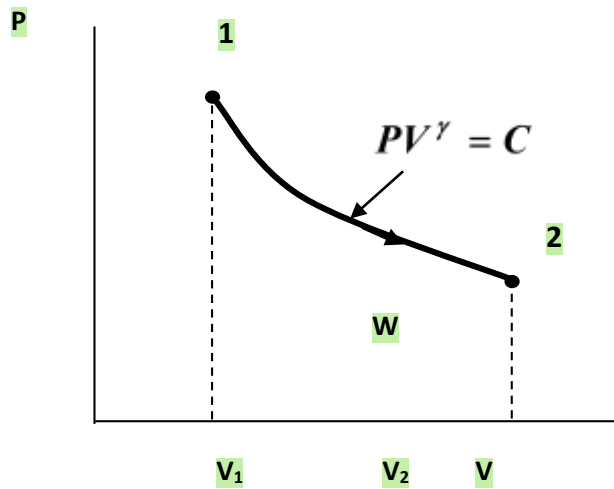


Fig (6-6) The Area Representing The Adiabatic Work

Equation (6-56) can be derived in another method depending on the adiabatic law.

$$W = \int_1^2 P dV$$

$$\therefore P V^\gamma = C \Rightarrow P = \frac{C}{V^\gamma}$$

$$\therefore W = \int_1^2 \frac{C}{V^\gamma} dV = C \int_1^2 \frac{dV}{V^\gamma}$$

$$W = C \left[\frac{V^{-\gamma+1}}{-\gamma+1} \right]_1^2 = C \left[\frac{V_2^{-\gamma+1} - V_1^{-\gamma+1}}{-\gamma+1} \right]$$

$$C = P_1 V_1^\gamma = P_2 V_2^\gamma$$

$$\therefore W = \left[\frac{P_2 V_2^\gamma V_2^{-\gamma+1} - P_1 V_1^\gamma V_1^{-\gamma+1}}{-\gamma + 1} \right]$$

After rearrangement the following equation is obtained.

$$W = \frac{P_1 V_1 - P_2 V_2}{\gamma - 1}$$

This equation is the same as equation (6-56).

Example 6-7

A quantity of gas with a mass of 1 kg expanded reversibly and adiabatically where its temperature dropped from 240 °C to 115 °C while its volume doubled. The work done during the process is 90 kJ. Calculate the specific heats of the gas c_p and c_v and its molecular weight.

Since the gas undergoes a reversible adiabatic non-flow process the adiabatic relations are used in the solution.

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2} \right)^{\gamma-1}$$

$$\frac{V_1}{V_2} = \frac{1}{2}$$

$$\therefore \frac{388}{513} = \left(\frac{1}{2} \right)^{\gamma-1}$$

$$\ln \left(\frac{388}{513} \right) = (\gamma - 1) \ln \left(\frac{1}{2} \right)$$

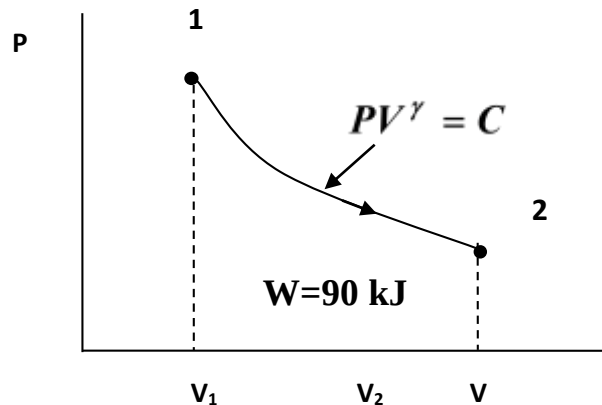
$$\therefore \gamma = 1.4026$$

$$W = \frac{mR(T_1 - T_2)}{\gamma - 1}$$

$$\therefore 90 \times 10^3 = \frac{1 \times R \times (513 - 388)}{1.4026 - 1}$$

$$\therefore R = 289.87 \text{ J/kg.K}$$

$$c_v = \frac{R}{\gamma - 1} = \frac{289.87}{1.4026 - 1} = 719.99 \text{ J/k.K}$$



$$c_p = \gamma c_v = 1.4026 \times 719.99 = 1009.866 \text{ J/kg.K}$$

5-The Polytropic Process

This process is represented mathematically by the following equation,

$$PV^n = C \quad (6-57)$$

This equation is known as the polytropic law.

All real non-flow processes for vapor and gas can be represented by this law depending on the value of the index (n) which is known as the polytropic index.

Fig (6-7) shows the process on the P-V diagram.

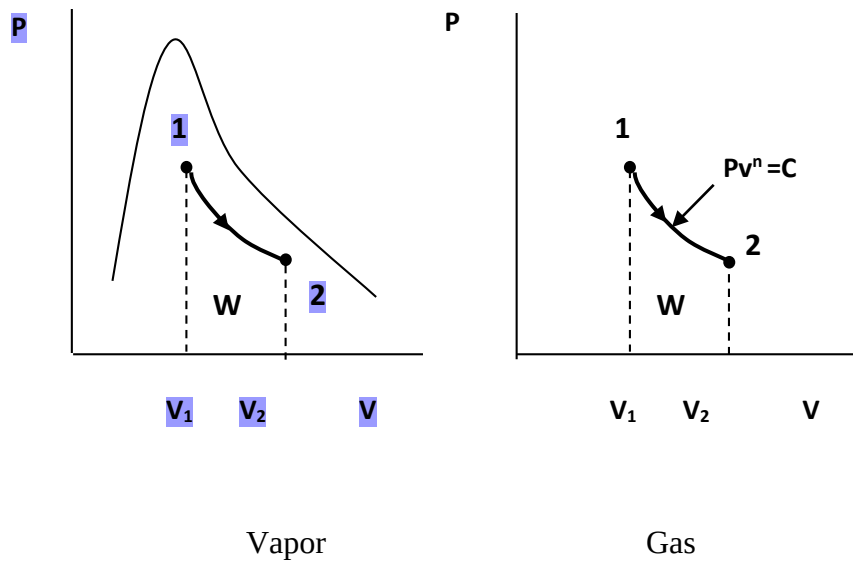


Fig (6-7) The Polytropic Process

Similar to the adiabatic ratios we can write the polytropic ratios as follows.

$$P_1 v_1^n = P_2 v_2^n = C$$

$$\frac{P_1}{P_2} = \left(\frac{v_2}{v_1} \right)^n \quad (6-58)$$

$$\frac{T_1}{T_2} = \left(\frac{v_2}{v_1} \right)^{n-1} \quad (6-59)$$

$$\frac{T_1}{T_2} = \left(\frac{P_1}{P_2} \right)^{\frac{n-1}{n}} \quad (6-60)$$

The equation for the work done during a polytropic process can be derived depending on the polytropic law and $W = \int_1^2 P dV$.

$$w = \frac{P_1 v_1 - P_2 v_2}{n-1} \quad (6-61)$$

For mass (m),

$$W = \frac{m(P_1 v_1 - P_2 v_2)}{n-1} = \frac{P_1 V_1 - P_2 V_2}{n-1} \quad (6-62)$$

Equations (6-61) and (6-62) are used to calculate the done for both vapor and gas if the value of the polytropic index is given.

A mathematical relation between the work done and the heat exchanged during the polytropic process of a perfect gas can be derived as follows starting with the non-flow energy equation

$$Q = m(u_2 - u_1) + W$$

$$Q = m(u_2 - u_1) + \frac{P_1 V_1 - P_2 V_2}{n-1}$$

$$Q = m c_v (T_2 - T_1) + \frac{m R (T_1 - T_2)}{n-1}$$

$$Q = m \left(c_v - \frac{R}{n-1} \right) (T_2 - T_1)$$

$$c_v = \frac{R}{\gamma - 1}$$

$$\therefore Q = m \left(\frac{R}{\gamma - 1} - \frac{R}{n-1} \right) (T_2 - T_1)$$

After simplification and rearrangement,

$$Q = \left(\frac{n - \gamma}{\gamma - 1} \right) \left[\frac{m R (T_2 - T_1)}{n - 1} \right]$$

$$Q = \left(\frac{\gamma - n}{\gamma - 1} \right) \left[\frac{m R (T_1 - T_2)}{n - 1} \right]$$

$$\therefore Q = \left(\frac{\gamma - n}{\gamma - 1} \right) W \quad (6-63)$$

The polytropic law may be considered as a general law to represent all non-flow processes depending on the value of the index (n) as shown in table (6-1) and fig (6-8).

Table (6-1)

Type of Process	Value of (n)	The Law
Constant Volume Process	$n = \infty$	$Pv^\infty = C, v = C$
Constant Pressure Process	$n = 0$	$Pv^0 = C, P = C$
Isothermal process	$n = 1$	$Pv = C$
Adiabatic Process	$n = \gamma$	$Pv^\gamma = C$
Polytropic Process	$n = n$	$Pv^n = C$

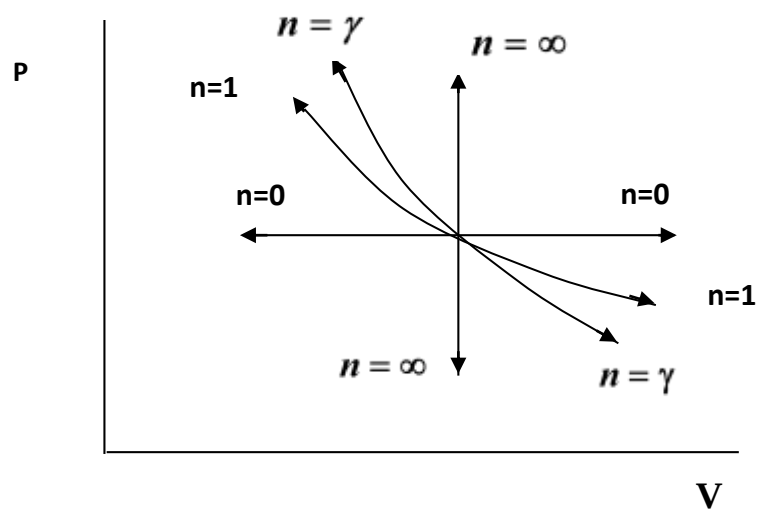


Fig (6-8)

Example 6-8

A wet steam with a dryness fraction of 0.9 expanded in a cylinder according to the law $Pv^{1.1} = C$ from a pressure of 2.5 bar. At the end of expansion process the volume of steam becomes four times its initial volume. Calculate the heat exchanged per unit mass of steam during the process.

This is a non-flow process. Apply the energy equation for non-flow processes.

$$q = (u_2 - u_1) + w$$

Find the properties of wet steam at initial state.

$$u_1 = u_f + x(u_g - u_f) = 535 + 0.9 \times (2537 - 535) = 2336.8 \text{ kJ/kg}$$

Since it is a polytropic process,

$$P_1 v_1^{1.1} = P_2 v_2^{1.1}$$

$$v_2 = 4v_1$$

$$\therefore P_2 = P_1 \left(\frac{v_1}{v_2} \right)^{1.1} = 2.5 \times \left(\frac{1}{4} \right)^{1.1} = 0.544 \text{ bar}$$

$$v_1 = x v_g = 0.9 \times 0.72 = 0.648 \text{ m}^3 / \text{kg}$$

$$v_2 = 4v_1 = 4 \times 0.648 = 2.592 \text{ m}^3 / \text{kg}$$

Since $v_2 < v_g$ at 0.544 bar the steam remains wet at end of process.

$$\therefore x = \frac{v_2}{v_g} = \frac{2.592}{2.96} = 0.876$$

$$\therefore u_2 = u_f + x(u_g - u_f) = 350 + 0.876 \times (2487 - 350) = 2222 \text{ kJ/kg}$$

$$w = \frac{P_1 v_1 - P_2 v_2}{n - 1}$$

$$\therefore w = \left\{ \frac{2.5 \times 10^5 \times 0.648 - 0.544 \times 10^5 \times 2.592}{1.1 - 1} \right\} \times 10^{-3} = 210 \text{ kJ/kg}$$

$$\therefore q = (u_2 - u_1) + w = (2222 - 2336.8) + 210 = 95.2 \text{ kJ/kg}$$

Heat is supplied since it is positive.

Example 6-9

Consider an amount of air with a volume of 0.3 m^3 at a pressure of 1 bar and a temperature of 26°C . The air is compressed according to the law $Pv^n = C$ until its volume becomes 0.07 m^3 .

Calculate the work done, the change in internal energy and the heat exchanged during the process.

The process is a polytropic reversible non-flow compression process.

$$\therefore P_1 V_1^{1.5} = P_2 V_2^{1.5}$$

$$\therefore P_2 = P_1 \left(\frac{V_1}{V_2} \right)^{1.5} = 1 \times \left(\frac{0.3}{0.07} \right)^{1.5} = 8.87 \text{ bar}$$

$$T_2 = T_1 \left(\frac{V_1}{V_2} \right)^{n-1} = 299 \times \left(\frac{0.3}{0.07} \right)^{1.5-1} = 619 \text{ K}$$

$$W = \frac{P_1 V_1 - P_2 V_2}{n-1}$$

$$\therefore W = \frac{1 \times 10^5 \times 0.3 - 8.87 \times 10^5 \times 0.07}{1.5-1} = -0.6418 \times 10^5 \text{ J} = -64.18 \text{ kJ}$$

$$m = \frac{P_1 V_1}{RT_1} = \frac{1 \times 10^5 \times 0.3}{287 \times 299} = 0.349 \text{ kg}$$

$$U_2 - U_1 = mc_v(T_2 - T_1) = 0.349 \times 0.718 \times (619 - 299) = 80.32 \text{ kJ}$$

$$Q = (U_2 - U_1) + W = 80.32 + (-64.18) = 16.14 \text{ kJ}$$

The Irreversible Non-flow Processes

As mentioned earlier all actual processes are completely irreversible and all mathematical formulation derived before are inapplicable. In this section some of these processes is considered.

1-Irreversible adiabatic process

As in the reversible adiabatic process there is no heat exchange with the surroundings ($Q=0$) during this process but the work done cannot be calculated as ($W = \int P dV$) since this equation is used for reversible processes only. However the non-flow energy equation is applicable.

$$Q = (U_2 - U_1) + W$$

$$\therefore Q = 0$$

$$\therefore W = U_1 - U_2 = m(u_1 - u_2) \quad (6-64)$$

The values of the internal energies u_1 and u_2 depend on the type of working fluid and its states initially and finally. It can be concluded from equation (6-64) that during this process the work done equals the change in internal energy.

2-The irreversible constant volume, constant pressure, constant temperature and polytropic processes

In all these processes the type of work fluid and its initial and final states should be known in addition to either the work done or the heat exchange. The non-flow energy equation is applicable.

The work done cannot be estimated using the equation ($W = \int PdV$).

$$Q = (U_2 - U_1) + W$$

$$Q - W = (U_2 - U_1)$$

3- Free or Un-resisted Expansion

This is the most irreversible process due to the violent swirling and turbulent motion that occurs as the fluid expanded. As in other irreversible processes the equation of work ($W = \int PdV$) cannot be used. The process can be explained by considering a vessel divided into two parts by a partition. One part contains a fluid at a pressure P_1 while the other is completely evacuated as shown in fig (6-9).

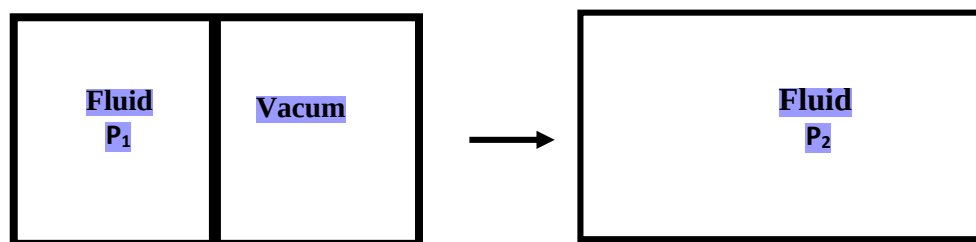


Fig (6-9) Free Expansion Process

Assume that the partition is destroyed and the fluid expanded suddenly to fill the whole vessel (free expansion) and finally reaches a state of thermal equilibrium at a pressure P_2 . The specific volume of the fluid becomes v_2 . The process can be assumed adiabatic ($Q=0$) since it occurs very quickly and no work is done ($W=0$). The non-flow energy equation can be applied.

$$Q = (U_2 - U_1) + W$$

$$\text{Since } Q = 0, W = 0$$

$$\therefore U_1 = U_2 \quad (6-65)$$

This means that there is no change in internal energy of working fluid during the free expansion process.

For vapor the internal energy is obtained from tables according to its quality and properties. For perfect gas the internal energy is obtained by applying Joules law.

$$c_v T_1 = c_v T_2$$

$$\therefore T_1 = T_2 \quad (6-66)$$

Equation (6-66) indicates that the free expansion process of a perfect gas is isothermal process.

Example 6-10

A vessel is divided into two parts. One part contains 1 kg of water at 700 kPa and 65 °C. The other part is empty. The partition is destroyed suddenly so that the water expanded to fill the whole vessel and the pressure becomes 15 kPa. Calculate the final temperature of water and the total volume of vessel.

At 700 kPa and 65 °C the water is compressed liquid initially undergoes a free expansion process.

The properties of compressed water are obtained from the compressed water tables.

$$\therefore u_1 = 271.6 \text{ kJ/kg}, \quad v_1 = 0.00102 \text{ m}^3/\text{kg}$$

$$\text{Since it is a free expansion process } \therefore Q = 0, \quad W = 0$$

$$\therefore u_2 = u_1 = 271.6 \text{ kJ/kg}$$

From saturated steam table find u_g at 15 kPa. $u_g = 2448.9 \text{ kJ/kg}$

Since $u_2 < u_g$ the water is in state of wet steam. The dryness fraction is calculated as,

$$x = \frac{u_2 - u_f}{u_g - u_f} = \frac{271.6 - 226}{2448.9 - 226} = 0.0205$$

The temperature equals the saturation temperature at 15 kPa which is $t_2 = 54^\circ \text{C}$.

$$v_2 = x v_g = 0.0205 \times 10.02 = 0.2055 \text{ m}^3/\text{kg}$$

$$V = m v_2 = 1 \times 0.2055 = 0.2055 \text{ m}^3$$

This is the total volume of vessel.