



Ministry of Higher Education and Scientific Research

Hilla university

Collage of Engineering

Department of Medical Instrumentation Techniques

Engineering

Mathematics (II)

By:

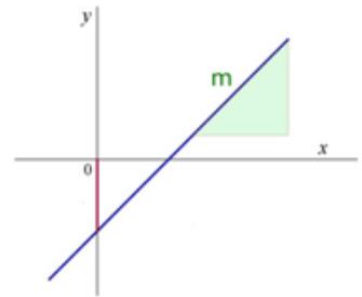
M.sc Huda Kadhim Rumuh

Line in space

4.1 Lines and Line Segments in Space

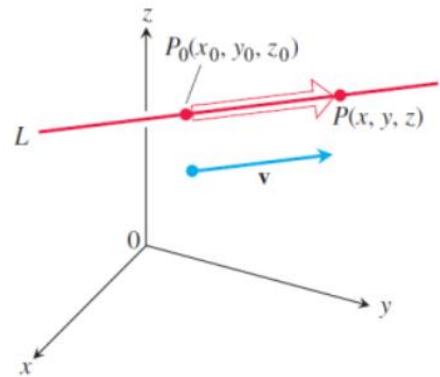
In the plane, a line is determined by a point and a number giving the slope of the line.

$$(y - y_1) = m(x - x_1)$$



In space a line is determined by a point and a vector giving the direction of the line.

Suppose that L is a line in space passing through a point $P_0 (x_0, y_0, z_0)$ parallel to a vector $\mathbf{v} = v_1 \mathbf{i} + v_2 \mathbf{j} + v_3 \mathbf{k}$.



$$L = \overrightarrow{PP_0}$$

$$\overrightarrow{PP_0} = t\mathbf{v}$$

$$(x - x_0)\mathbf{i} + (y - y_0)\mathbf{j} + (z - z_0)\mathbf{k} = t(v_1\mathbf{i} + v_2\mathbf{j} + v_3\mathbf{k})$$

$$xi + yj + zk = x_0i + y_0j + z_0k + t(v_1i + v_2j + v_3k)$$

$$xi + yj + zk = x_0i + y_0j + z_0k + t(v_1i + v_2j + v_3k)$$

The equation can be written as:

$$r(t) = r_0(t) + tv, \quad -\infty < t < \infty \quad \textbf{Vector Equation for a Line}$$

A vector equation for the line L through $P_0(x_0, y_0, z_0)$ parallel to v is

where

r : is the position vector of a point $P(x, y, z)$ on L

r_0 is the position vector of $P_0(x_0, y_0, z_0)$

Equating the corresponding components of the two sides of the vector equation gives three scalar equations involving the parameter **t**:

$$xi + yj + zk = (x_0 + tv_1)i + (y_0 + tv_2)j + (z_0 + tv_3)k$$

➤ **Parametric Equations for a Line**

The standard parametrization of the line through $P_0(x_0, y_0, z_0)$ parallel to $\mathbf{v} = v_1\mathbf{i} + v_2\mathbf{j} + v_3\mathbf{k}$:

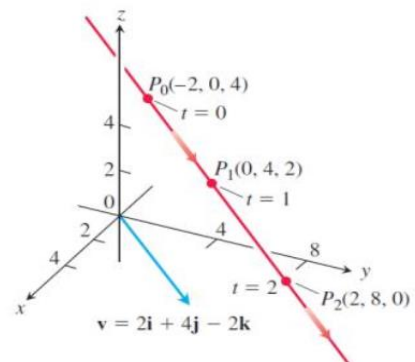
$$x = x_0 + tv_1, \quad y = y_0 + tv_2, \quad z = z_0 + tv_3 \quad -\infty < t < \infty$$

Example: Find parametric equations for the line through $(-2, 0, 4)$ parallel to $\mathbf{v} = 2\mathbf{i} + 4\mathbf{j} - 2\mathbf{k}$

sol:

$$x = x_0 + tv_1, \quad y = y_0 + tv_2, \quad z = z_0 + tv_3$$

$$x = -2 + 2t, \quad y = 4t, \quad z = 4 - 2t$$



Example: Find parametric equations for the line through $\mathbf{P}(-3, 2, -3)$ and $\mathbf{Q}(1, -1, 4)$.

Sol:

$$\text{The vector: } \vec{PQ} = (1 - (-3))\mathbf{i} + (-1 - 2)\mathbf{j} + (4 - (-3))\mathbf{k} = 4\mathbf{i} - 3\mathbf{j} + 7\mathbf{k}$$

$$(x_0, y_0, z_0) = (-3, 2, -3)$$

$$x = -3 + 4t, \quad y = 2 - 3t, \quad z = -3 + 7t$$

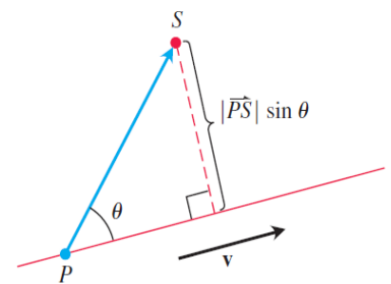
4.2 The Distance from a Point to a Line in Space

Distance from a Point S to a Line Through P Parallel to $\mathbf{v}$

scalar component is $|\vec{PS}| \sin \theta$, which is $\frac{|\vec{PS}| |\mathbf{v}| \sin \theta}{|\mathbf{v}|} = \frac{|\vec{PS} \times \mathbf{v}|}{|\mathbf{v}|}$.

$$|\vec{PS}| |\mathbf{v}| \sin \theta = |\vec{PS} \times \mathbf{v}|$$

$$d = \frac{|\vec{PS} \times \mathbf{v}|}{|\mathbf{v}|}$$



Example: Find the distance from the point $S(1, 1, 5)$ to the line $L: x = 1 + t, y = 3 - t, z = 2t$.

Sol:

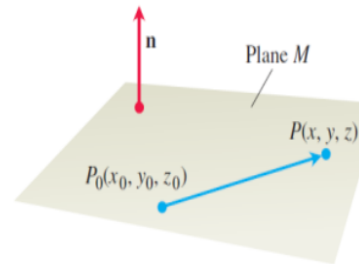
We see from the equations for L that L passes through $P(1, 3, 0)$ parallel to $\mathbf{v} = \mathbf{i} - \mathbf{j} + 2\mathbf{k}$.

$$\overrightarrow{PS} = (1 - 1)\mathbf{i} + (1 - 3)\mathbf{j} + (5 - 0)\mathbf{k} = -2\mathbf{j} + 5\mathbf{k}$$

$$\overrightarrow{PS} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & -2 & 5 \\ 1 & -1 & 2 \end{vmatrix} = \mathbf{i} + 5\mathbf{j} + 2\mathbf{k} \quad d = \frac{|\overrightarrow{PS} \times \mathbf{v}|}{|\mathbf{v}|} = \frac{\sqrt{1 + 25 + 4}}{\sqrt{1 + 1 + 4}} = \frac{\sqrt{30}}{\sqrt{6}} = \sqrt{5}$$

4.3 Planes in Space

- ✓ Suppose that plane M passes through a point $P_0(x_0, y_0, z_0)$ and is normal to the nonzero vector $\mathbf{n} = A\mathbf{i} + B\mathbf{j} + C\mathbf{k}$.
- ✓ A vector from P_0 to any point P on the plane is orthogonal to $\mathbf{n}$
- ✓ Then M is the set of all points $P(x, y, z)$ for which $\overrightarrow{P_0P}$ is orthogonal to $\mathbf{n}$
- ✓ Thus, the dot product $\mathbf{n} \cdot \overrightarrow{P_0P} = 0$.
- ✓ Thus $(A\mathbf{i} + B\mathbf{j} + C\mathbf{k}) \cdot [(x - x_0)\mathbf{i} + (y - y_0)\mathbf{j} + (z - z_0)\mathbf{k}] = 0$
- ✓ So the plane M consists of the points (x, y, z) satisfying:



$$A(x - x_0) + B(y - y_0) + C(z - z_0) = 0$$

Equation for a Plane

The plane through $P_0(x_0, y_0, z_0)$ normal to $\mathbf{n} = A\mathbf{i} + B\mathbf{j} + C\mathbf{k}$ has

Vector equation: $\mathbf{n} \cdot \overrightarrow{P_0P} = 0$

Component equation: $A(x - x_0) + B(y - y_0) + C(z - z_0) = 0$

Component equation simplified: $Ax + By + Cz = D$, where $D = Ax_0 + By_0 + Cz_0$

Example: Find an equation for the plane through $P_0(-3, 0, 7)$ perpendicular to $\mathbf{n} = 5\mathbf{i} + 2\mathbf{j} - \mathbf{k}$.

Sol: The component equation is

$$A(x - x_0) + B(y - y_0) + C(z - z_0) = 0 \quad , \quad 5(x - (-3)) + 2(y - 0) + (-1)(z - 7) = 0.$$

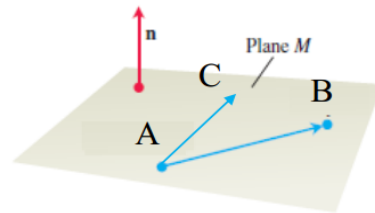
$$5x + 15 + 2y - z + 7 = 0$$

$$5x + 2y - z = -22$$

Example: Find an equation for the plane through $A(0, 0, 1)$, $B(2, 0, 0)$, and $C(0, 3, 0)$.

Sol:

$$\vec{AB} \times \vec{AC} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & 0 & -1 \\ 0 & 3 & -1 \end{vmatrix} = 3\mathbf{i} + 2\mathbf{j} + 6\mathbf{k}$$

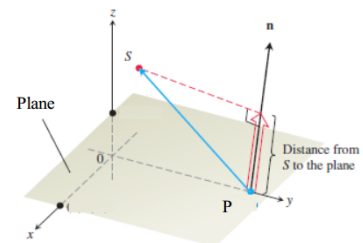


$$3(x - 0) + 2(y - 0) + 6(z - 1) = 0 \quad , \quad 3x + 2y + 6z = 6$$

4.4 Distance from a Point to a Plane

➤ The Distance from a Point to a Plane

If P is a point on a plane with normal $\mathbf{n}$, then the distance from any point S to the plane is the length of the vector projection of $\overrightarrow{PS}$ onto $\mathbf{n}$, as given in the following formula.



Distance from a Point S to a Plane with Normal $\mathbf{n}$ at Point P :

$$d = \left| \overrightarrow{PS} \cdot \frac{\mathbf{n}}{\|\mathbf{n}\|} \right|$$

Example: Find the distance from $S(1, 1, 3)$ to the plane $3x + 2y + 6z = 6$ where $P(0,3,0)$

Sol:

The coefficients in the equation $3x + 2y + 6z = 6$ give: $\mathbf{n} = 3\mathbf{i} + 2\mathbf{j} + 6\mathbf{k}$.

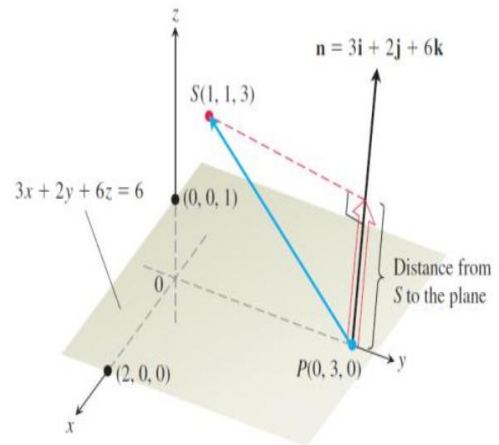
$$\overrightarrow{PS} = (1 - 0)\mathbf{i} + (1 - 3)\mathbf{j} + (3 - 0)\mathbf{k} = \mathbf{i} - 2\mathbf{j} + 3\mathbf{k}$$

$$|\mathbf{n}| = \sqrt{(3)^2 + (2)^2 + (6)^2} = \sqrt{49} = 7.$$

$$d = \left| \overrightarrow{PS} \cdot \frac{\mathbf{n}}{|\mathbf{n}|} \right|$$

$$= \left| (\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}) \cdot \left(\frac{3}{7}\mathbf{i} + \frac{2}{7}\mathbf{j} + \frac{6}{7}\mathbf{k} \right) \right|$$

$$= \left| \frac{3}{7} - \frac{4}{7} + \frac{18}{7} \right| = \frac{17}{7}.$$



Example: Given the plane with equation $7x + 2y - 3z = 1$, find a normal vector to the plane and identify three points that lie on that plane.

Sol: A possible normal vector is:

$$\mathbf{n} = 7\mathbf{i} + 2\mathbf{j} - 3\mathbf{k}.$$

Finding three points in the plane is not difficult. First, let $y = z = 0$ in the defining equation and solve for x :

$$7x + 2 \cdot 0 - 3 \cdot 0 = 1 \iff 7x = 1 \iff x = \frac{1}{7} \quad \text{Thus } \left(\frac{1}{7}, 0, 0\right) \text{ is a point on the plane}$$

Next, let $x = z = 0$ and solve for y :

$$7 \cdot 0 + 2y - 3 \cdot 0 = 1 \iff y = \frac{1}{2} \quad \text{So } \left(0, \frac{1}{2}, 0\right) \text{ is another point on the plane}$$

Finally, let $x = y = 0$ and solve for z

You should find that $\left(0, 0, -\frac{1}{3}\right)$ lies on the plane

Assignment

I. Find parametric equations for the lines

1. The line through the origin parallel to the vector $2\mathbf{j} + \mathbf{k}$
2. The line through the point $(3, -2, 1)$ parallel to the line $x = 1 + 2t$,
 $y = 2 - t$, $z = 3t$
3. The line through $(1, 1, 1)$ parallel to the z -axis
4. The line through $(2, 4, 5)$ perpendicular to the plane $3x + 7y - 5z = 21$

II. Find the distance from the point to the line.

1. $(2, 1, -1)$; $x = 2t$, $y = 1 + 2t$, $z = 2t$
2. $(3, -1, 4)$; $x = 4 - t$, $y = 3 + 2t$, $z = -5 + 3t$
3. $(-1, 4, 3)$; $x = 10 + 4t$, $y = -3$, $z = 4t$

III. Find equations for the planes

1. The plane through $(1, -1, 3)$ parallel to the plane $3x + y + z = 7$
2. The plane through $(2, 4, 5)$, $(1, 5, 7)$, and $(-1, 6, 8)$

IV. Find the distance from the point to the plane.

1. $(2, 2, 3)$, $2x + y + 2z = 4$
2. $(0, -1, 0)$, $2x + y + 2z = 4$
3. $(1, 0, -1)$, $-4x + y + z = 4$